Collaboration of Heterogeneous Multi-Agent Systems with Terrain-Dependent Mobility

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Course: MAE 276 Geometric Nonlinear Control

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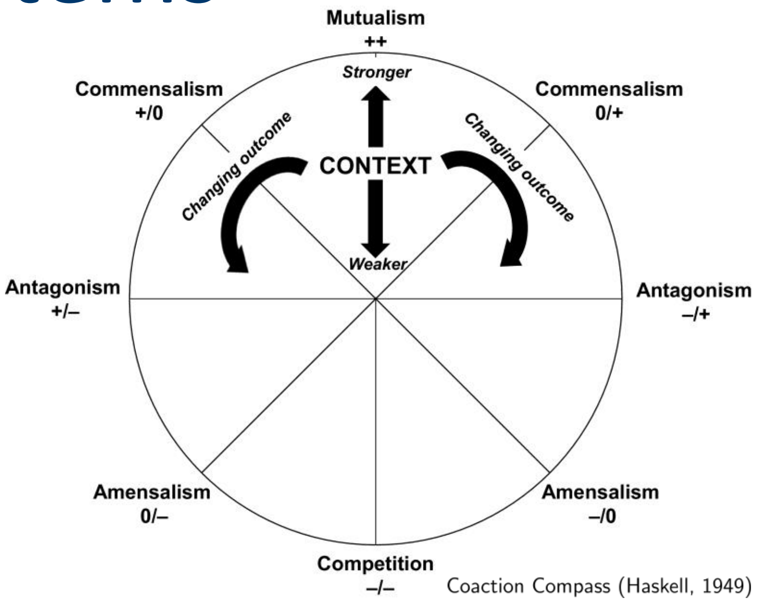
(Biological) Multi-Agent Systems

"Jointly beneficial interactions between members of different species."

Pauli et al., 2015, *Proc. R. Soc. B.*



Coaction Compass (Haskell, 1949)

https://www.nhm.ac.uk/discover/mutualism-examples-of-species-that-work-together.html

Pistol Shrimps and Gobies

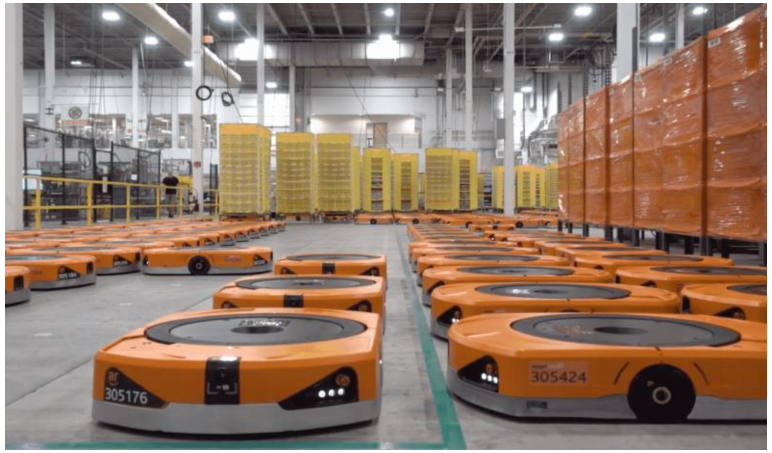
https://www.nhm.ac.uk/discover/mutualism-examples-of-species-that-work-together.html

Honeyguides and Humans

https://www.zoho.com/blog/recruit/what-recruiters-can-learn-from-crocodiles-and-plovers.html

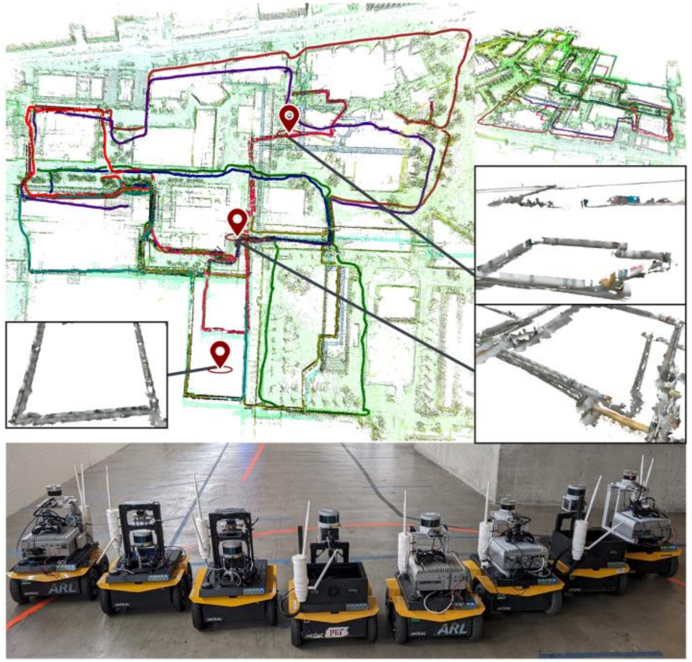
Nile Crocodile and Egyptian Plover

(Engineered) Multi-Agent Systems



https://www.waredock.com/magazine/what-is-amazon-robotic-fulfillment-center/

Amazon Warehouses



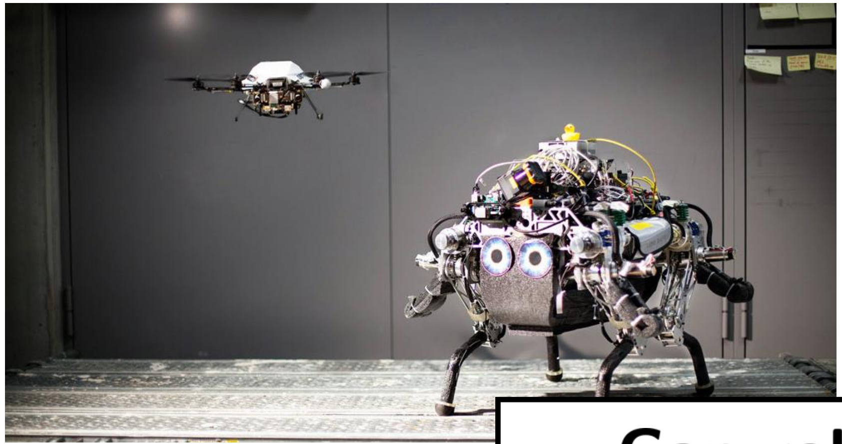
https://spectrum.ieee.org/multi-robot-slam-icra2023

Localization and Mapping

https://www.aglaw.us/janzenaglaw/2020/7/16/is-your-farm-ready-for-the-swarm

Agriculture Processes

Heterogeneity



https://robohub.org/air-and-ground-robot-collaborate-to-map-and-safe

Mobility-Based Opera

moon.html

imitations

Can robot heterogeneity be exploited for collaboration?

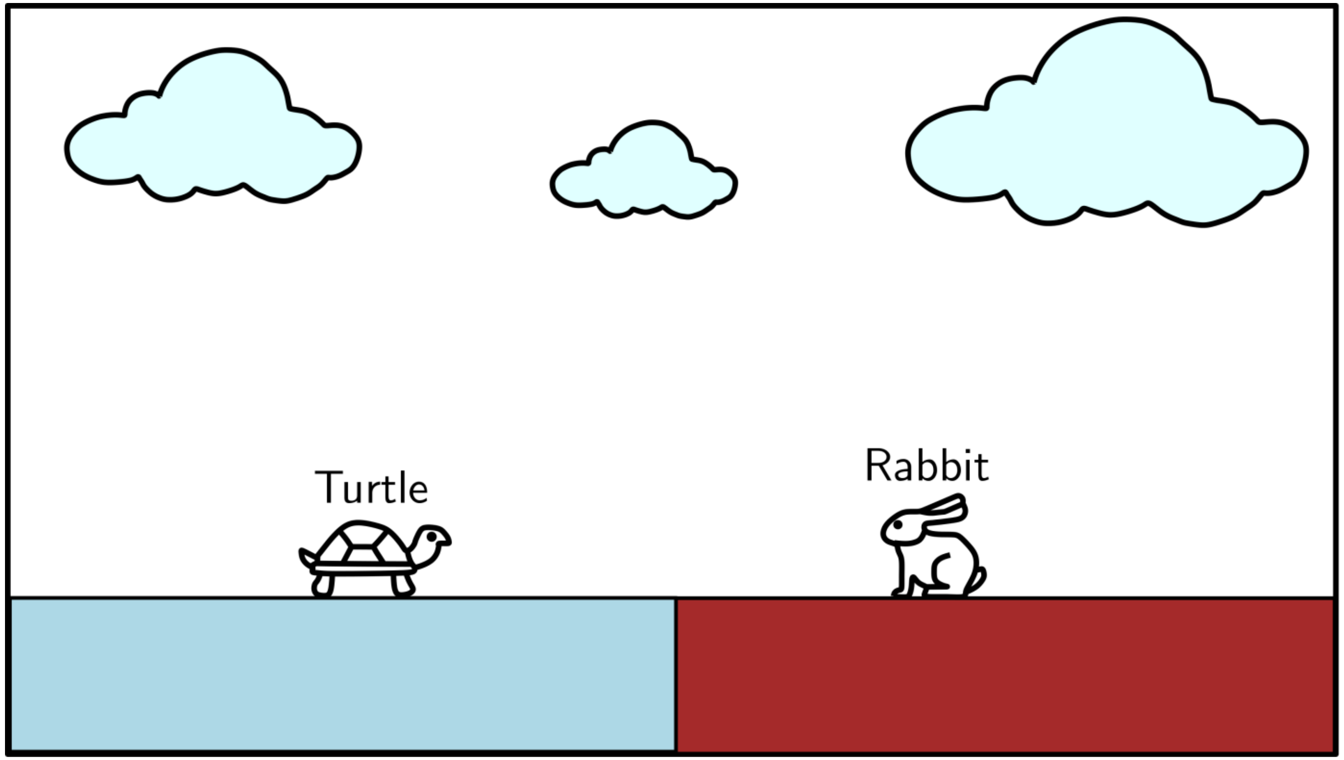
https://www.idtechex.com/en/research-article/idtechex-outlines-the-future-of-the-agricultural-robotics-industry/25744

Body Shapes

https://www.frontiersin.org/articles/10.3389/fnbot.2020.576846/full

Sensing Modalities

Agents in a Shared Workspace



Consider two agents coexisting within a shared workspace

- Amphibious
- Mobility depends on terrain

Rabbit in pond

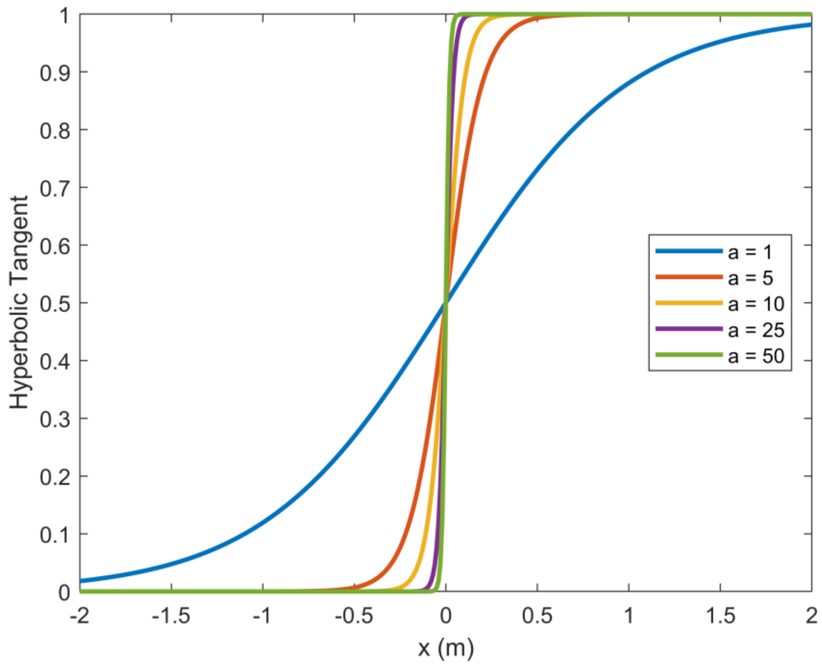
Turtle on land

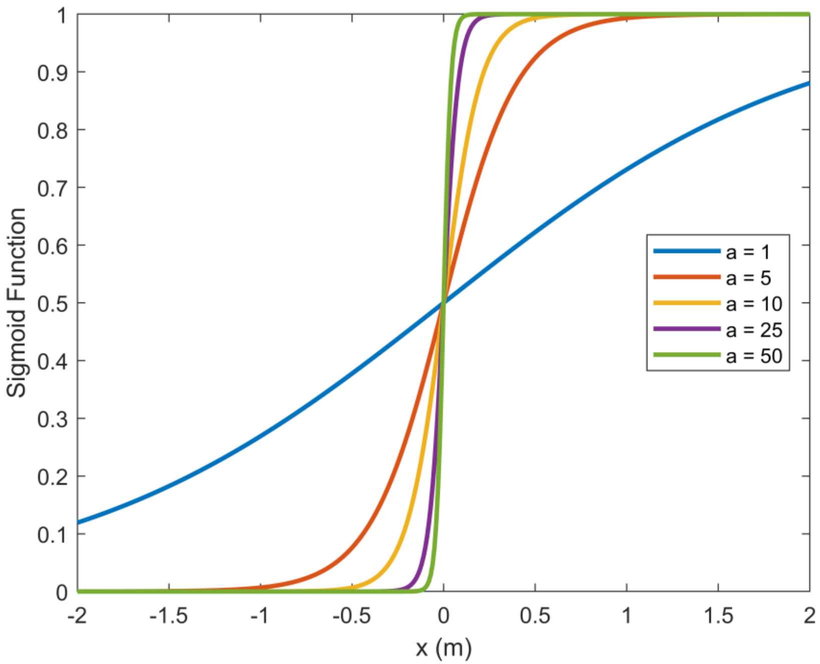
Q: What about if the rabbit and turtle worked together??

Terrain-Dependent Mobility Gain

Consider the terrain-dependent gain of agent i to be a function of spatial x position only, i.e., $\kappa_r(x_r)$ and $\kappa_t(x_t)$.

Two candidate functions which can capture the desired behavior are





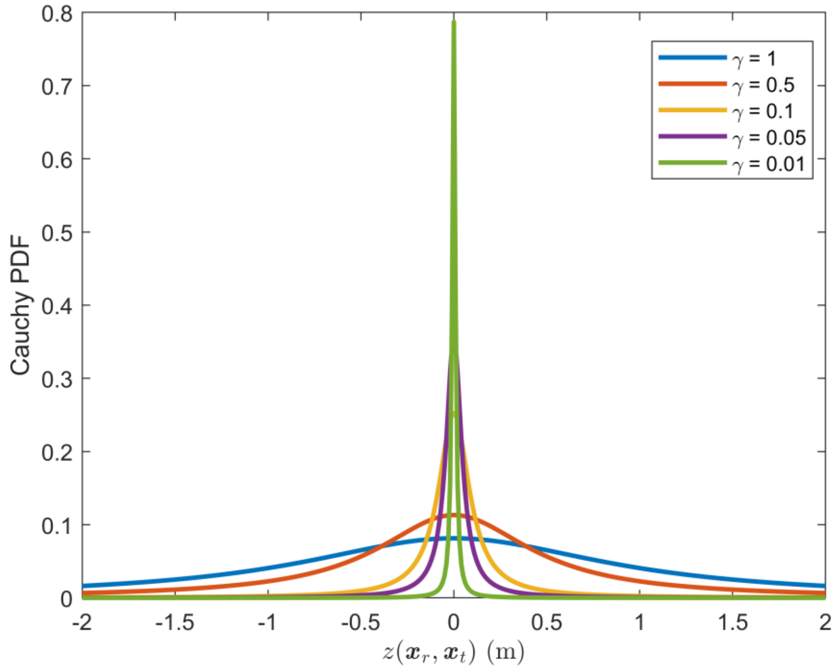
Hyperbolic Tangent: $\kappa(x) = \frac{1}{2}(\tanh(ax) + 1)$ Sigmoid Function: $\kappa(x) = \frac{1}{1+\exp(-ax)}$

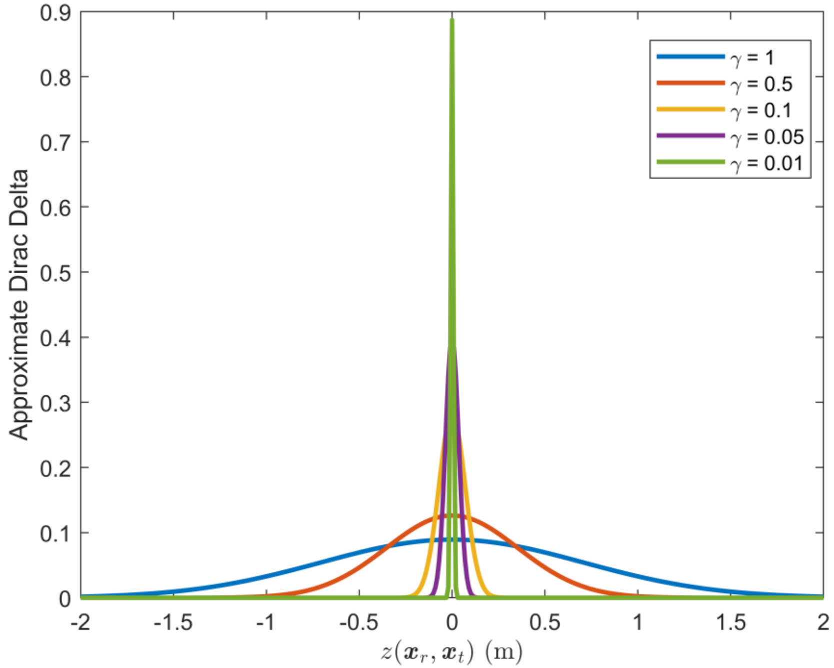
Remark: $\kappa_t(x_t) = 1 - \kappa_r(x_t)$ (e.g., $\kappa_r(x_r) = \frac{1}{1+\exp(-ax_r)}$, then $\kappa_t(x_t) = 1 - \frac{1}{1+\exp(-ax_t)}$)

Proximity Metric

Consider a proximity metric to distinguish the relative distance between agents to be a function of planar position states, i.e., $\chi(\boldsymbol{x}_r, \boldsymbol{x}_t)$ where $\boldsymbol{x}_r = [x_r, y_r]^{\mathsf{T}}$ and $\boldsymbol{x}_t = [x_t, y_t]^{\mathsf{T}}$.

Two candidate functions which can capture the desired behavior are





Cauchy PDF: $\chi(\boldsymbol{x}_r, \boldsymbol{x}_t) = \frac{1}{\pi} \frac{\gamma}{(z(\boldsymbol{x}_r, \boldsymbol{x}_t))^2 + \gamma^2}$

Approx. Dirac Delta: $\chi(\boldsymbol{x}_r, \boldsymbol{x}_t) = \frac{1}{\gamma\sqrt{\pi}} \exp\left[-\left(\frac{z(\boldsymbol{x}_r, \boldsymbol{x}_t)}{\gamma}\right)^2\right]$

Remark: We can normalize $\chi(\boldsymbol{x}_r, \boldsymbol{x}_t)$ to one, and have $z(\boldsymbol{x}_r, \boldsymbol{x}_t) = \|\boldsymbol{x}_r - \boldsymbol{x}_t\|_2^2$.

System Dynamics

We will consider dynamics in control affine form, i.e., $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \sum_j \boldsymbol{g}_j(\boldsymbol{x})\boldsymbol{u}_j$, given as

$$\underbrace{\begin{bmatrix} \dot{\boldsymbol{x}}_r \\ \dot{\boldsymbol{x}}_t \\ \dot{\alpha} \end{bmatrix}}_{\dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} G_r(x_r) \\ \alpha\chi(\boldsymbol{x}_r, \boldsymbol{x}_t) \cdot I_{d\times d} \\ 0 \end{bmatrix}}_{\boldsymbol{g}_r(\boldsymbol{x})} \boldsymbol{u}_r + \underbrace{\begin{bmatrix} \alpha\chi(\boldsymbol{x}_r, \boldsymbol{x}_t) \cdot I_{d\times d} \\ G_t(x_t) \\ 0 \end{bmatrix}}_{\boldsymbol{g}_t(\boldsymbol{x})} \boldsymbol{u}_t + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \kappa \end{bmatrix}}_{\boldsymbol{g}_{\text{collab}}} v,$$

where

- $d = \{1, 2, 3\}$ is the position states' dimension
- $G_r(x_r) = \kappa_r(x_r) \cdot I_{d\times d} \succ 0$ is the rabbit's control gain
- $G_t(x_t) = \kappa_t(x_t) \cdot I_{d\times d} \succ 0$ is the turtle's control gain
- $\chi(\boldsymbol{x}_r, \boldsymbol{x}_t) \geq 0$ is the proximity metric quantifying the closeness between agents
- $\kappa > 0$ is the control gain corresponding to collaboration strength
- $\boldsymbol{x}_r \in \mathbb{R}^d, \boldsymbol{x}_r \in \mathbb{R}^d$ are the rabbit and turtle (position) states, respectively
- α distinguishes the difference between collaboration ($\alpha \neq 0$) and closeness ($\alpha = 0$)

Several Probing Questions

For simplicity, we will consider the case of $d = 1$ (1-D position states), e.g., $\boldsymbol{x}_r \in \mathbb{R}$ and $\boldsymbol{x}_t \in \mathbb{R}$. Now, we will pose the following questions:

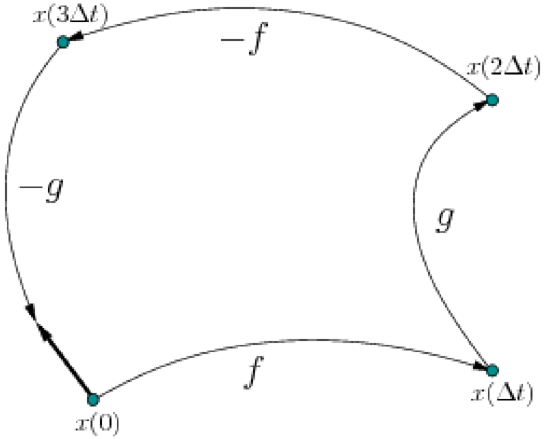
Q1: What happens when we start taking Lie brackets with our controlled dynamics vector fields?

Q2: Will the Lie bracket's control gain be larger than the original control gain? If yes, under what conditions does this hold?

Q3: How can the control gain obtained from taking the Lie Bracket be realized?

Taking Lie Brackets of Vector Fields

Recall: the Lie bracket is defined as $[X,Y] = J_Y X - J_X Y$ where J_Y, J_X are $n \times n$ Jacobian matrices and X, Y are vector fields.



A1: <u>Assume:</u> $\kappa_r(x_r) = \frac{1}{2}(\tanh(ax_r)+1)$, $\kappa_t(x_t) = 1 - \frac{1}{2}(\tanh(ax_t)+1)$

$$\chi(x_r, x_t) = \frac{1}{\gamma\sqrt{\pi}} \exp\left[-\left(\frac{(x_r - x_t)^2}{\gamma}\right)^2\right]$$

(a) $[\boldsymbol{g}_t(\boldsymbol{x}), \boldsymbol{g}_{\text{collab}}] = J_{\boldsymbol{g}_{\text{collab}}} \boldsymbol{g}_t(\boldsymbol{x}) - J_{\boldsymbol{g}_t(\boldsymbol{x})} \boldsymbol{g}_{\text{collab}}$

$$= \begin{bmatrix} -\frac{\exp\frac{-(xr-xt)^2}{\gamma^2}}{\sqrt{\pi}\gamma} \\ 0 \\ 0 \end{bmatrix} \qquad \boldsymbol{g}_t(\boldsymbol{x}) = \begin{bmatrix} \alpha\frac{\exp\frac{-(xr-xt)^2}{\gamma^2}}{\sqrt{\pi}\gamma} \\ 1 - \frac{1}{2}(\tanh(a \cdot x_t) + 1) \\ 0 \end{bmatrix}$$

$\Rightarrow$ This Lie bracket does not help us gain more control authority.

· Similarily, we can compare $[\boldsymbol{g}_r(\boldsymbol{x}), \boldsymbol{g}_{\text{collab}}]$ and $\boldsymbol{g}_r(\boldsymbol{x})$, but the result will be the same.

Taking Lie Brackets of Vector Fields (Continued)

(b) $[\boldsymbol{g}_t(\boldsymbol{x}), \boldsymbol{g}_r(\boldsymbol{x})] = J_{\boldsymbol{g}_r(\boldsymbol{x})}\boldsymbol{g}_t(\boldsymbol{x}) - J_{\boldsymbol{g}_t(\boldsymbol{x})}\boldsymbol{g}_r(\boldsymbol{x})$

$$= \begin{bmatrix} \dfrac{\exp^{-\frac{2(x_r-x_t)^2}{\gamma^2}} \alpha \left(-4(x_r-x_t)\alpha + a\exp^{\frac{(x_r-x_t)^2}{\gamma^2}} \sqrt{\pi}\gamma^3 \mathrm{sech}^2(ax_r) + 2\exp^{\frac{(x_r-x_t)^2}{\gamma^2}} \sqrt{\pi}(x_r-x_t)\gamma(1+\tanh(ax_r))\right)}{2\pi\gamma^4} \\ \dfrac{\exp^{-\frac{2(x_r-x_t)^2}{\gamma^2}} \alpha \left(a\exp^{\frac{(x_r-x_t)^2}{\gamma^2}} \sqrt{\pi}\gamma^3 \mathrm{sech}^2(ax_t) - 2(x_r-x_t)\left(2\alpha - \exp^{\frac{(x_r-x_t)^2}{\gamma^2}} \sqrt{\pi}\gamma - \exp^{\frac{(x_r-x_t)^2}{\gamma^2}} \sqrt{\pi}\gamma \tanh(ax_t)\right)\right)}{2\pi\gamma^4} \\ 0 \end{bmatrix}$$

$$\boldsymbol{g}_t(\boldsymbol{x}) = \begin{bmatrix} \alpha \dfrac{\exp^{\frac{-(xr-xt)^2}{\gamma^2}}}{\sqrt{\pi}\gamma} \\ 1 - \frac{1}{2}(\tanh(a \cdot x_t) + 1) \\ 0 \end{bmatrix}$$

$\Rightarrow$ This Lie bracket has the potential to help us gain more control authority!

Realization of Improved Control Authority

A2: Improved control authority should only happen when $\chi(x_r, x_t) \neq 0$, so we assume $x_r = x_t$

$$\Rightarrow [\boldsymbol{g}_t(\boldsymbol{x}),\boldsymbol{g}_r(\boldsymbol{x})] = \begin{bmatrix} \frac{a\alpha \mathrm{sech}^2(ax_t)}{2\sqrt{\pi}\gamma} \\ \frac{a\alpha \mathrm{sech}^2(ax_t)}{2\sqrt{\pi}\gamma} \\ 0 \end{bmatrix}, \ \boldsymbol{g}_t(\boldsymbol{x}) = \begin{bmatrix} \frac{\alpha}{\sqrt{\pi}\gamma} \\ 1 - \frac{1}{2}(\tanh(ax_t)+1) \\ 0 \end{bmatrix}, \ \boldsymbol{g}_r(\boldsymbol{x}) = \begin{bmatrix} \frac{1}{2}(\tanh(ax_t)+1) \\ \frac{\alpha}{\sqrt{\pi}\gamma} \\ 0 \end{bmatrix}$$

For the turtle, let us determine when control gain would be larger for the Lie bracket vector field

$$\frac{a\alpha \mathrm{sech}^2(ax_t)}{2\sqrt{\pi}\gamma} \geq 1 - \frac{1}{2}(\tanh(ax_t)+1) \quad \rightarrow \quad \left[\frac{\alpha}{\gamma}\right]_t \geq \frac{2\sqrt{\pi}(1-\frac{1}{2}(\tanh(ax_t)+1))}{a\mathrm{sech}^2(ax_t)} = \frac{2\sqrt{\pi}\kappa_t(x_t)}{a\mathrm{sech}^2(ax_t)}$$

For the rabbit, let us determine when control gain would be larger for the Lie bracket vector field

$$\frac{a\alpha \mathrm{sech}^2(ax_r)}{2\sqrt{\pi}\gamma} \geq \frac{1}{2}(\tanh(ax_r)+1) \quad \rightarrow \quad \left[\frac{\alpha}{\gamma}\right]_r \geq \frac{2\sqrt{\pi}(\frac{1}{2}(\tanh(ax_r)+1))}{a\mathrm{sech}^2(ax_r)} = \frac{2\sqrt{\pi}\kappa_r(x_r)}{a\mathrm{sech}^2(ax_r)}$$

as long as this inequality hold, the Lie bracket can improve control authority!

A3: Improved control authority (i.e., higher gain) may be achievable by flowing along the Lie bracket direction

$$\phi_{\sqrt{t}}^{-\boldsymbol{g}_r(\boldsymbol{x})} \circ \phi_{\sqrt{t}}^{-\boldsymbol{g}_t(\boldsymbol{x})} \circ \phi_{\sqrt{t}}^{\boldsymbol{g}_r(\boldsymbol{x})} \circ \phi_{\sqrt{t}}^{\boldsymbol{g}_t(\boldsymbol{x})}$$

Optimal Control Formulation

$$\min_{\boldsymbol{x}(\cdot),\ \boldsymbol{u}(\cdot)} \quad J = \int_0^{t_f} \|\boldsymbol{u}(t)\|_2^2 \, dt$$ → Control Energy (Cost Function)

s.t.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{g}_r(\boldsymbol{x}(t))\boldsymbol{u}_r(t) + \boldsymbol{g}_t(\boldsymbol{x}(t))\boldsymbol{u}_t(t) + \boldsymbol{g}_{\text{collab}} v(t)$$ → System Dynamics (Constraint)

$$\boldsymbol{x}(0) = \boldsymbol{x}_0, \ \boldsymbol{x}(t_f) = \boldsymbol{x}_f$$ → Boundary Conditions (Constraint)

$$x_{\min} \le x_r(t) \le x_{\max}, \ x_{\min} \le x_t(t) \le x_{\max}$$
$$y_{\min} \le y_r(t) \le y_{\max}, \ y_{\min} \le y_t(t) \le y_{\max}$$ → Boxed Domain (Constraint)

$$\|\boldsymbol{u}(t)\|_\infty \le \bar{u},$$ → Bounding Actuation (Constraint)

where

$$\boldsymbol{x}_r(t) \in \mathcal{X}_r \subset \mathbb{R}^2, \ \boldsymbol{x}_t(t) \in \mathcal{X}_t \subset \mathbb{R}^2, \ \alpha \in \mathbb{R}$$
$$\boldsymbol{x}(t) = [\boldsymbol{x}_r(t), \boldsymbol{x}_t(t), \alpha(t)]^\mathsf{T} \in \mathcal{D} \subset \mathbb{R}^5$$
$$\boldsymbol{u}(t) = [\boldsymbol{u}_r(t), \boldsymbol{u}_t(t), v(t)]^\mathsf{T} \in \mathcal{U} \subset \mathbb{R}^5$$

Q: How to solve this optimal control problem?

$\Rightarrow$ Using an open-source software tool for numerical optimization and optimal control!

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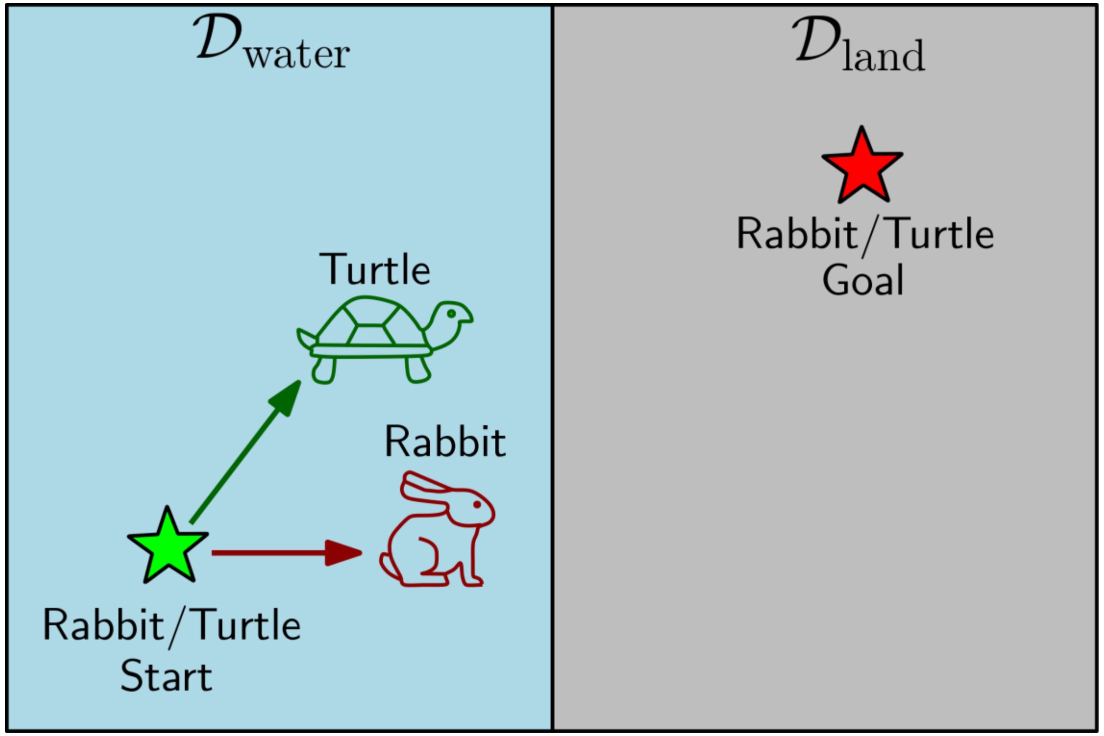
Simulation Settings

Consider the 2-D domain to be compact, and defined as $\mathcal{D} = \mathcal{D}_{\text{water}} \cup \mathcal{D}_{\text{land}}$



Simulation Settings:

- Time Horizon = 30 s
- Sampling Time = 0.1 s
- $\mathcal{D} = [-2, 2] \times [-2, 2]$ $([x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}])$
- $\|u(t)\|_\infty \leq \bar{u} = 1$ m/s
- IC : $x_0 = x_r(0) = x_t(0) = [-1, -1]^\mathsf{T}$
- FC : $x_f = x_r(t_f) = x_t(t_f) = [1, 1]^\mathsf{T}$
- Individual: $\alpha(t) = 0 \ \forall t$
- Collaboration: $\alpha(0) = \alpha(t_f) = 0$ } To Activate Collaboration

Terrain-Dependent Proportional Gains:

$$\kappa_r(x_r) = \frac{1}{2}(\tanh(a x_r) + 1) \in [0.05, 1]$$

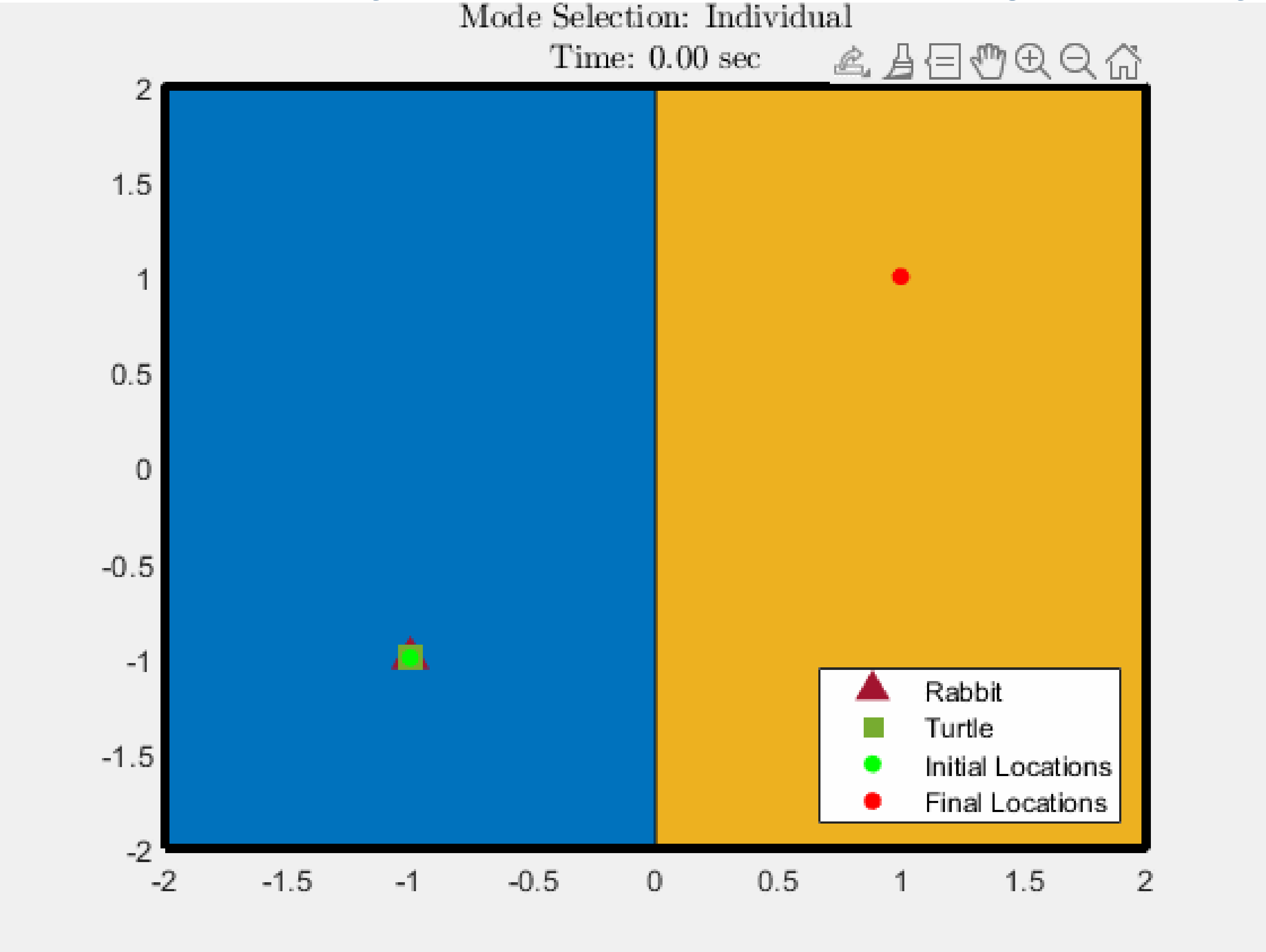
$$\kappa_t(x_t) = 1 - \frac{1}{2}(\tanh(a x_t) + 1) \in [0.05, 1]$$

Proximity Function:

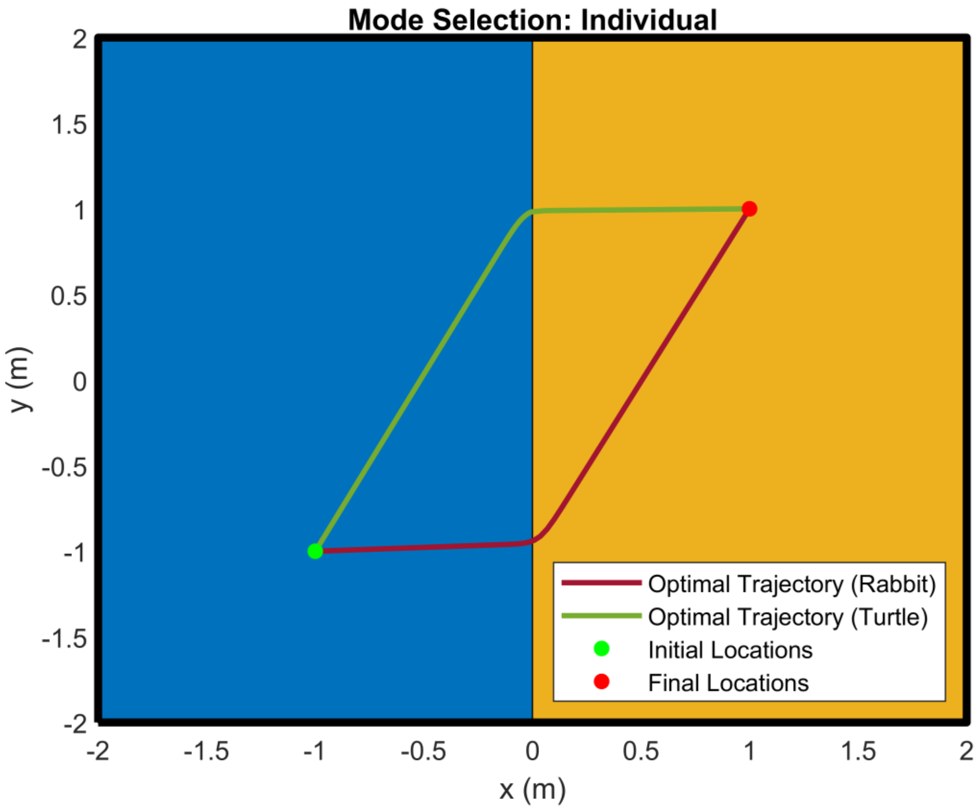
Let $\chi'(\boldsymbol{x}_r, \boldsymbol{x}_t) = \frac{1}{\gamma\sqrt{\pi}} \exp\left[-\left(\frac{z(\boldsymbol{x}_r, \boldsymbol{x}_t)}{\gamma}\right)^2\right]$

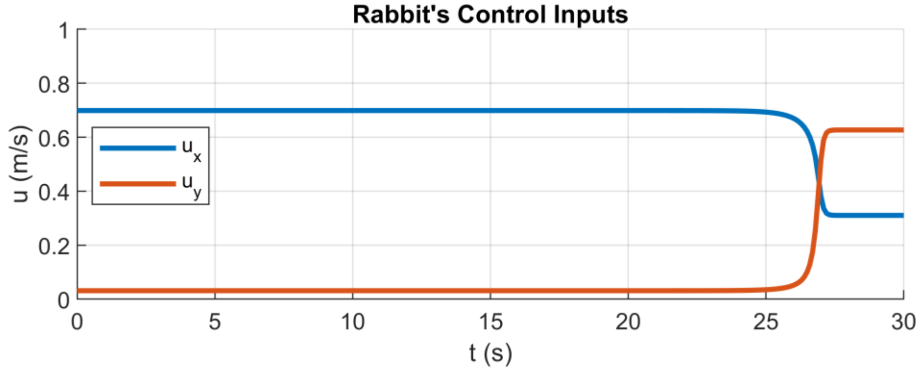
$$\rightarrow \chi(\boldsymbol{x}_r, \boldsymbol{x}_t) = \frac{\chi'(\boldsymbol{x}_r, \boldsymbol{x}_t)}{\|\chi'(\boldsymbol{x}_r, \boldsymbol{x}_t)\|_2}$$

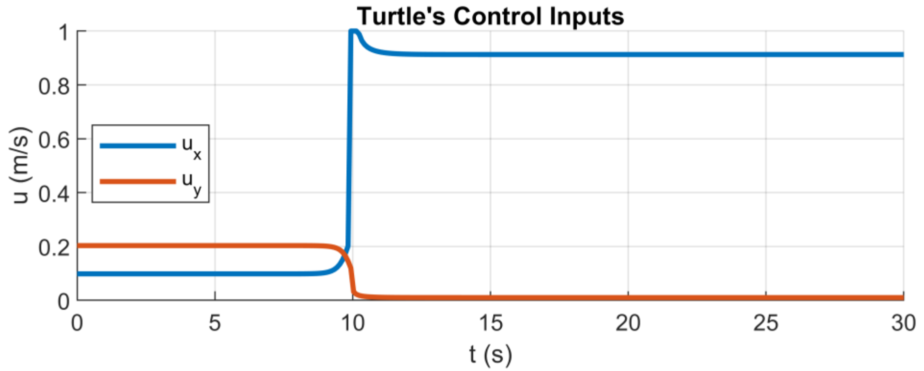
Case Study 1: Individual Trajectory



Case Study 1: Individual Trajectory







Optimization Solver Run-Time = 6.76 seconds

Simulation Time = 30 seconds

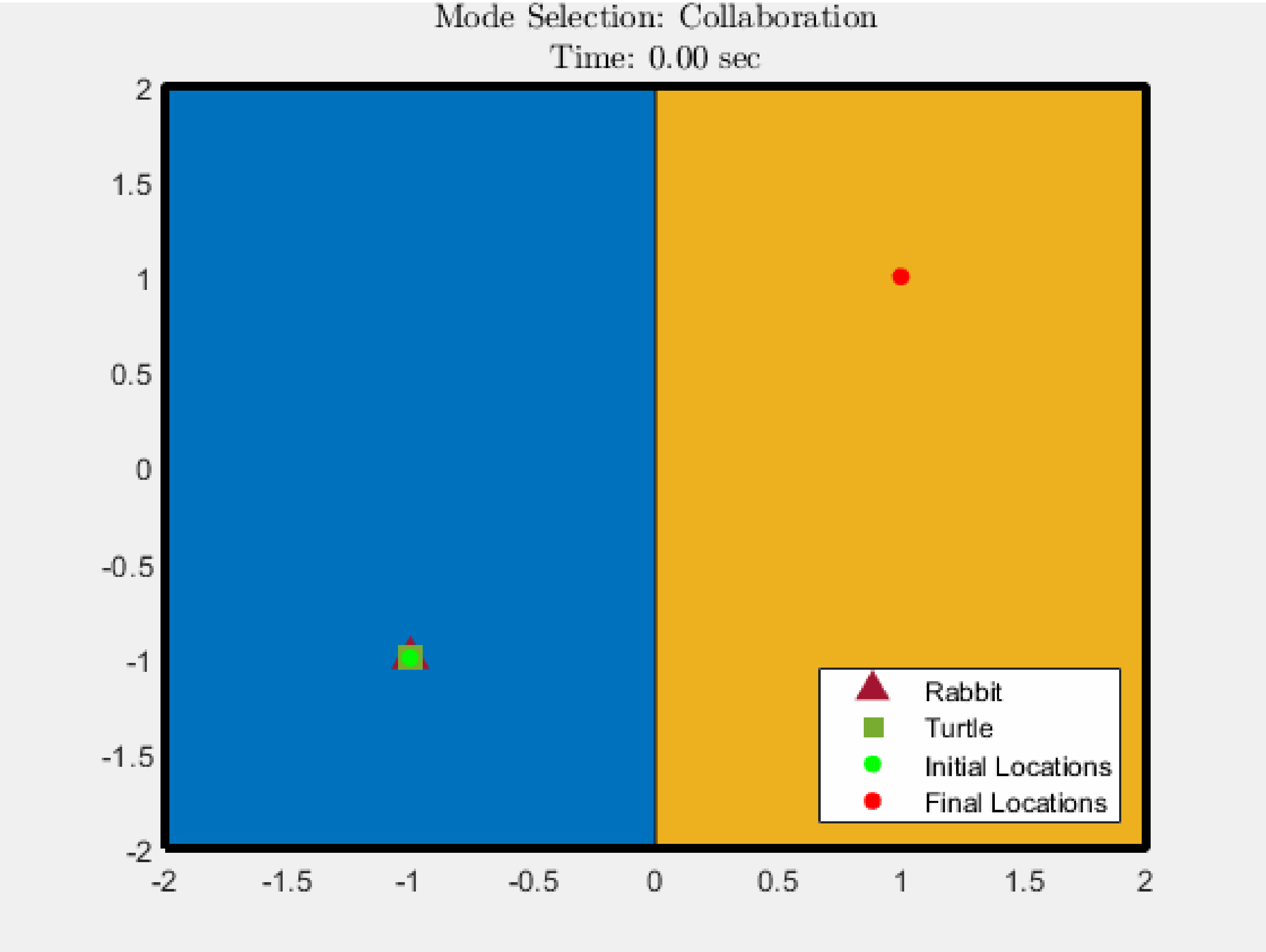
Rabbit Energy = 145.54 J

Turtle Energy = 155.71 J

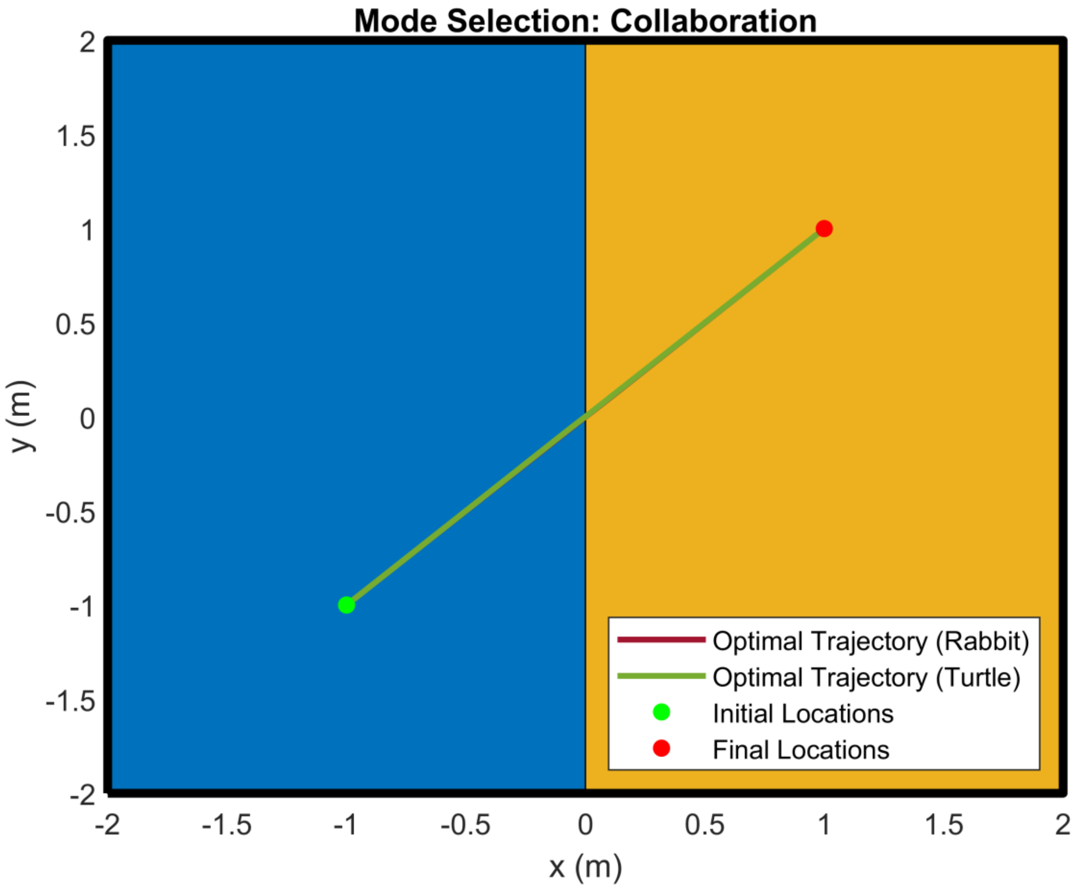
Collab. Activation Energy = 0 J

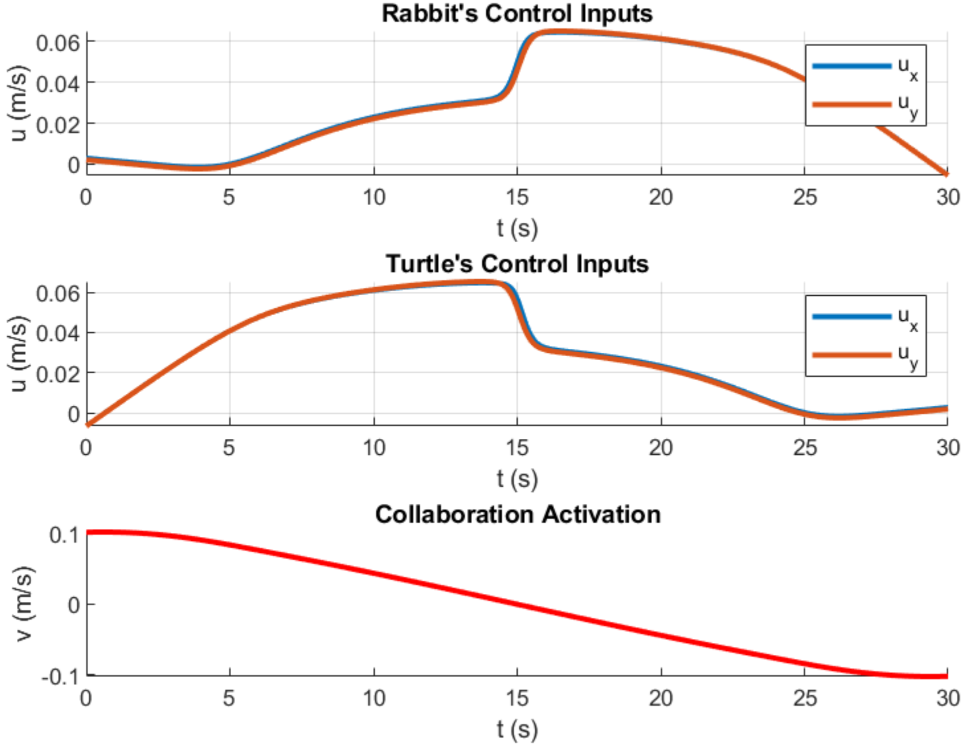
Total = 301.24 J

Case Study 2: Collaborative Trajectory



Case Study 2: Collaborative Trajectory



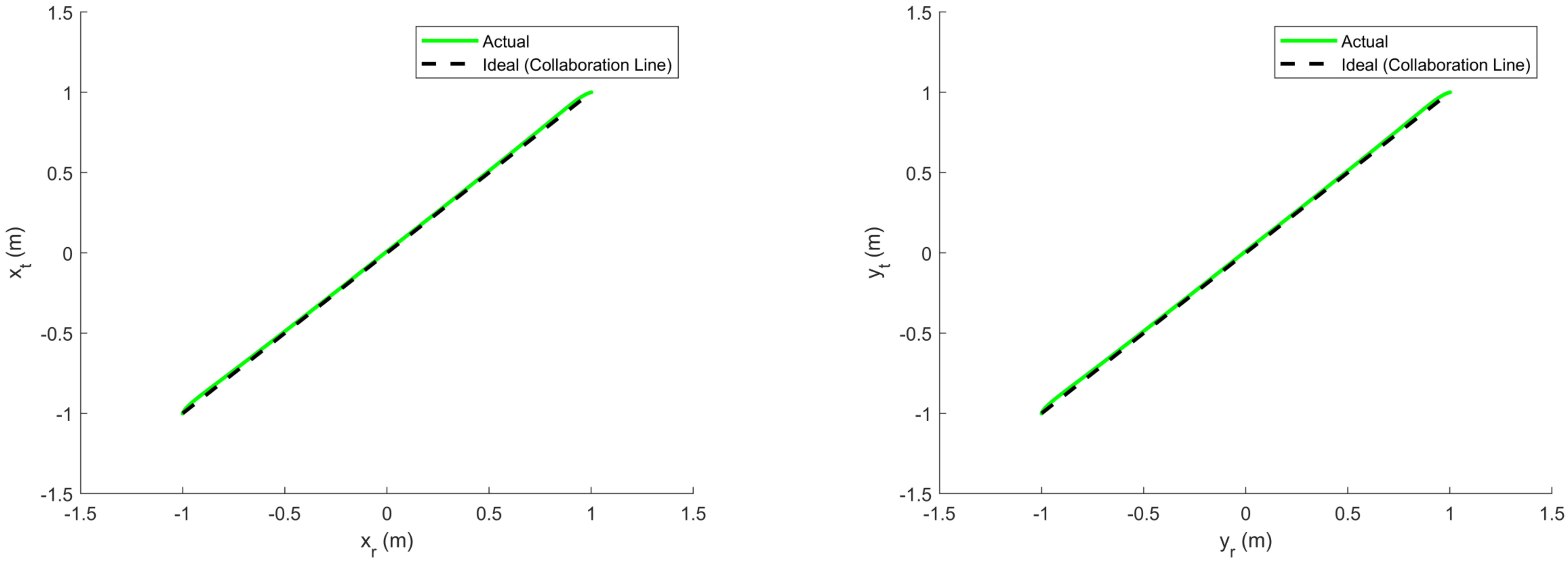


$$\text{Optimization Solver Run-Time} = 11.96 \text{ seconds}$$

$$\text{Simulation Time} = 30 \text{ seconds}$$

$$\left.\begin{aligned} \text{Rabbit Energy} &= 0.85 \text{ J} \\ \text{Turtle Energy} &= 0.85 \text{ J} \\ \text{Collab. Activation Energy} &= 1.44 \text{ J} \end{aligned}\right\} \text{Total} = 3.14 \text{ J}$$

Case Study 2: Collaborative Trajectory (Continued)



Collaboration is feasible when the position states lie on the characteristic lines of $x_r(t) = x_t(t)$ and $y_r(t) = y_t(t)$!

Thank you for listening!