MAE 276: Geometric Nonlinear Control	Spring Quarter 2023
Final Project Proposal	
Instructor: Prof. Haithem Taha	*Student: Alex Nguyen*

1 Project Background

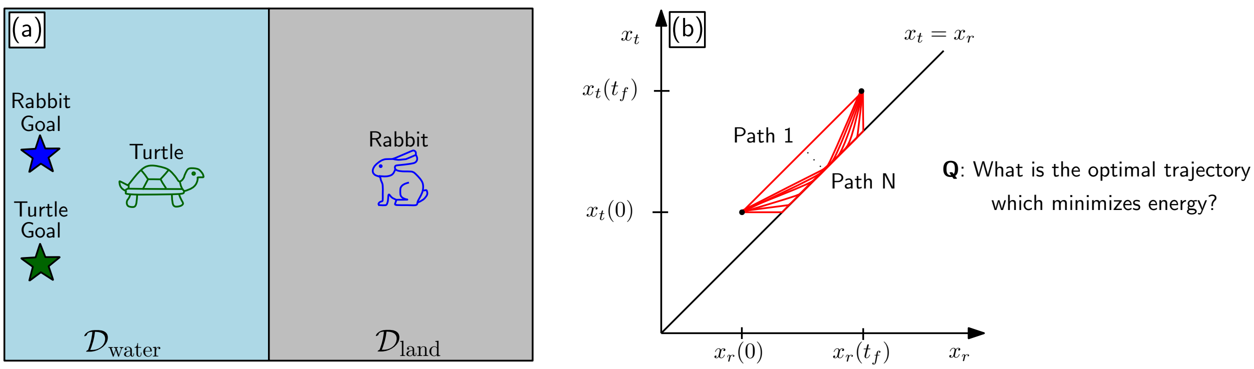

Figure 1: (a) An illustrative scenario that highlights the potential need for collaboration between two amphibious robots, pictorially shown as a rabbit and turtle. (b) Visual representation of the position states given by different path trajectories (shown in red). The separatrix, denoted as $x_r = x_t$, indicates the subset of the shared workspace domain, $\mathcal{D} = \mathcal{D}_{\text{water}} \cup \mathcal{D}_{\text{land}}$, where collaboration is feasible.

This project aims to investigate the energy expenditure of heterogeneous robots, in the context of terrain-dependent mobility, with navigation tasks. We focus on two amphibious robots, referred to as a "rabbit" and a "turtle", which will operate in a convex and compact workspace, as depicted in Figure 1. The rabbit is better suited to traverse land-based subdomains, $\mathcal{D}_{\text{land}}$, whereas the turtle is better suited to traverse water-based subdomains, $\mathcal{D}_{\text{water}}$. For instance, the rabbit consumes less energy when moving within the land subdomain compared to the water subdomain.

We will consider the robots to be modeled in a control affine form, which is commonly observed for many robotic systems, by use of single integrator dynamics, which is given as

$$\underbrace{\begin{bmatrix} \dot{x}_r \\ \dot{x}_t \\ \dot{\alpha} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} G_r(x_r) \\ \alpha\chi(x_r, x_t) \\ 0 \end{bmatrix}}_{g_r(x)} u_r + \underbrace{\begin{bmatrix} \alpha\chi(x_r, x_t) \\ G_t(x_t) \\ 0 \end{bmatrix}}_{g_t(x)} u_t + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \kappa \end{bmatrix}}_{g_{\text{collab}}} v,$$

where $x = [x_r, x_t, \alpha]^\top$ is the state vector consisting of the planar position states of rabbit, denoted as $x_r = [p_{r,x}, p_{r,y}]^\top$, and turtle, denoted as $x_t = [p_{t,x}, p_{t,y}]^\top$, and α is a state variable that triggers collaboration between the robots which happens only when $\dot{\alpha} \neq 0$. Moreover, u_r, u_t are the control inputs for the rabbit and turtle robots, respectively, while v is a control input that activates a collaborative endeavor between the two robots. Additionally, $G_r(x_r)$, $G_t(x_t)$ are positive gain matrices that are dependent on the terrain and position of the rabbit and turtle, respectively, and

$\chi(x_r, x_t)$ is a non-negative gain matrix that evaluates the similarity between the position states of the rabbit and turtle. This $\chi(x_r, x_t)$ function is used to verify the feasibility of collaboration, i.e., check if the robots' states are located on the separatrix illustrated in Figure 1(b). Lastly, κ is a positive gain constant necessary to initiate collaboration.

It is evident that the rabbit possesses larger control authority on land, while the turtle has higher control authority in water.[1] For example, if the rabbit were to enter the water subdomain, it would expend more energy compared to its heterogeneous counterpart, i.e., the turtle. However, the collaboration between the rabbit and the turtle may offer an opportunity to enhance their control authority. Therefore, this project aims to investigate whether it is possible to improve the control strength of both robots when taking Lie brackets between the vector fields governing the controlled dynamics, i.e., $g_r(x)$, $g_t(x)$, and g_{collab}.

Subsequently, following the study of the effect of collaboration on the robots' control authority, we will attempt to find a solution to the following optimal control problem

$$
\begin{aligned}
\min_{u_r(\cdot),\ u_t(\cdot)} \quad & J = \int_0^{t_f} \left(\|u_r(t)\|_2^2 + \|u_t(t)\|_2^2\right) dt \\
\text{s.t.} \quad & \dot{x}(t) = g_r(x(t))u_r(t) + g_t(x(t))u_t(t) + g_{\text{collab}}v(t) && \text{(dynamics)} \\
& x_r(t) \in \mathcal{X},\ x_t(t) \in \mathcal{X} && \text{(admissible state values)} \\
& u_r(t) \in \mathcal{U},\ u_t(t) \in \mathcal{U}, && \text{(admissible control inputs)} \\
& x(0) = x_0,\ x(t_f) = x_f && \text{(initial and final boundary conditions)}
\end{aligned}
$$

where the optimal solution is defined as the trajectory that minimizes energy while satisfying all constraints, i.e., dynamics, states, control inputs, and boundary conditions.[2]

2 Project Objectives

- Examine the impact of taking the Lie bracket between the vector fields governing the controlled dynamics. How does this affect the magnitude of the robots' control inputs?
  - (a) Utilize different gain matrices, namely $G_r \succ 0$, $G_t \succ 0$, and $\chi \succeq 0$, and change the magnitude of the gain constant $\kappa > 0$.[3]
  - (b) Let a condition for collaboration be defined as $\|x_t - x_r\|_2 \leq \delta_{\text{tol}}$. What happens when the distance tolerance between robots is varied (e.g., $\delta_{\text{tol}} = \{10^{-6}, 10^{-2}, 0.5\}$)?
  - (c) How can the robots realize their new control efforts?
- Solve the optimal control problem using numerical software, such as ICLOCS (free), CasADi (free), or GPOPS (not free). Does the optimal trajectory involve a collaborative endeavor?
  - (a) Plot the optimal trajectory, control inputs, x_t versus x_r, and the cost function over the desired time horizon.

[1] Greater control authority implies exerting less energy while attempting to track desired velocities, whereas lower control authority implies exerting more energy.

[2] Although, a straight line represents the shortest path in a Euclidean space, the optimal trajectory may exhibit some curvature as illustrated in Figure 1(b).

[3] Notation: $\succ$ is positive definite; $\succeq$ is positive semi-definite; $>$ is positive scalar.