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Final Project Proposal

Project Description

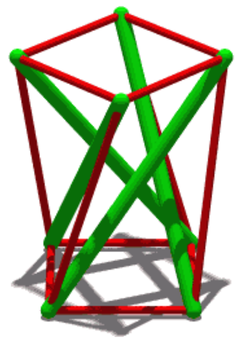

Figure 1: Tensegrity structure with springs (red lines) and rigid rods (green lines).

We consider the tensegrity structure illustrated in Figure 1, where the red lines represent springs, which have suitable spring constants, that can either be compressed or extended and the green lines represent rigid rods, which are difficult to compress, that have fixed edge lengths.

This final project aims to find an embedding for a graph, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, in $\mathbb{R}^3$ (as we would like to consider a physical mechanical structure) that minimizes the "energy" stored in the springs, given by

$$E_{\text{spring}} = \text{trace}(L_{\text{spring}} X X^\mathsf{T}),$$

where $X \in \mathbb{R}^{n \times 3}$ specifies the positioning of the corresponding n nodes such that

$$X = \begin{bmatrix} x_{11} & y_{12} & z_{13} \\ \vdots & \vdots & \vdots \\ x_{n1} & y_{n2} & z_{n3} \end{bmatrix},$$

with the k-th row of X being $x_k = [x_{k1}, y_{k2}, z_{k3}]^\mathsf{T}$, which provides the x-, y-, and z-position states of node k. On the other hand, the rigid rods have a fixed distance, given by

$$\|x_i - x_j\|_2^2 = d_{ij}, \ \forall i, j \in \mathcal{V}_{\text{rod}},$$

where $d_{ij} \geq 0$ is the rigid rod's fixed distances and $\mathcal{V}_{\text{rod}}$ is the index set of nodes for the rigid rods – whereas $\mathcal{V}_{\text{spring}}$ is the index set of nodes for the springs such that $\mathcal{V} = \mathcal{V}_{\text{rod}} \cup \mathcal{V}_{\text{spring}}$ – which can be imposed as a constraint. We can also impose the constraint $\mathbf{1}^\mathsf{T} X = 0$, such that the embedding of the graph is centered at the origin.

Hence, we can formulate a non-convex optimization problem, which is difficult to solve and may result in a non-unique solution depending on initialization, given as

$$\begin{aligned} \arg\min_{X} \quad & \text{trace}(L_{\text{spring}} X X^\mathsf{T}) \\ s.t. \quad & \|x_i - x_j\|_2^2 = d_{ij}, \quad \forall i, j \in \mathcal{V}_{\text{rod}}, \\ & \mathbf{1}^\mathsf{T} X = 0, \end{aligned}$$

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Project Tasks

- Perform a literature review of related problems/ideas;
- Verify that the formulated non-convex optimization problem is correct;
  - Are other constraints needed?
- Solve a numerical example with the adjacency matrices for the rigid rods and springs given as

$$A_{\text{rod}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \; A_{\text{spring}} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix},$$

  respectively;
- Summarize the findings of this project in a 3-4 slide presentation and a small report.