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Resiliency Through Collaboration in Heterogeneous Multi-Robot Systems

ALEXANDER A. NGUYEN [1] **(Graduate Student Member, IEEE), FARYAR JABBARI** [1] **(Life Senior Member, IEEE), AND MAGNUS EGERSTEDT** [1,2] **(Fellow, IEEE)**

(Resilient and Safe Control in Multi-Agent Systems)

[1]Department of Mechanical and Aerospace Engineering, University of California, Irvine, CA 92697 USA
[2]Department of Electrical Engineering and Computer Science, University of California, Irvine, CA 92697 USA

CORRESPONDING AUTHOR: ALEXANDER A. NGUYEN (e-mail: alexaan2@uci.edu).

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ABSTRACT This paper examines pairwise collaborations in heterogeneous multi-robot systems. In particular, we focus on how individual robots, with different functionalities and dynamics, can enhance their resilience by forming collaborative arrangements that result in new capabilities. Control barrier functions are utilized as a mechanism to encode the safe operating regions of individual robots, with the idea being that a robot may be able to operate in new regions that it could not traverse alone by working with other robots. We explore answers to three questions: "Why should robots collaborate?", "When should robots collaborate?", and "How can robots collaborate?" To that end, we introduce the safely reachable set – capturing the regions that individual robots can reach safely, either with or without help, while considering their initial states and dynamics. We then describe the conditions under which a help-providing robot and a help-receiving robot can engage in collaboration. Next, we describe the pairwise collaboration framework, modeled through hybrid automata, to show how collaborations can be structured within a heterogeneous multi-robot team. Finally, we present case studies that are conducted on a team of mobile robots.

INDEX TERMS Barrier functions, collaboration, heterogeneous multi-robot systems, reachable sets, and robot ecology.

I. INTRODUCTION

Multi-robot systems are commonly utilized to execute a primary mission in a coordinated and distributed manner [1]. For instance, heterogeneous multi-robot teams have been deployed for the applications of environmental monitoring [2], exploration [3], search and rescue [4], and transportation [5], such that they can accomplish complex objectives, which their homogeneous counterparts may have difficulties completing [6].

A standard approach in heterogeneous multi-robot systems is to separate the group into subteams based on their capability types, where each subteam is assigned a suitable task based on their skill set [7], [8]. This works well in some scenarios, but such an approach does not enable the individual robots to obtain new abilities that do not manifest themselves individually. Whereas, if a team of heterogeneous robots works together, they can potentially acquire new functionalities that a robot could not perform by itself, as observed in [9], [10], [11].

By working as a team, individual robots can become more "resilient". In this paper, resiliency is interpreted as when a heterogeneous robot team can work together, i.e., collaborate, safely and overcome what would otherwise be insurmountable obstructions, which could appear unannounced. In other words, individual robots can gain new capabilities to achieve objectives that a single robot cannot complete by itself [12]. For instance, environmental landscapes have the potential to be altered by external disturbances such as wildfire, landslide, or flooding. However, by having the robots form suitable collaborative arrangements, they can overcome such environmental changes, making the team of robots more resilient to external disturbances [13]. For example, an aerial robot could lift a small robot over impassable flames caused by a wildfire, or a suitably shaped robot could act as a ramp for ground robots unable to reach a high ledge created by a landslide. Thus, through collaboration, individual robots can become more resilient – i.e., continue to function in the presence of obstacles as they perform their assigned tasks.

The conceptualization of between-species collaboration is not new, especially within the field of ecology and evolution [14], [15], [16]. In nature, collaborative arrangements typically arise from evolutionary pressures on organisms over many years [17]. For example, mutualisms – jointly beneficial interactions between members of different species [18] – and commensalisms – unilaterally beneficial interactions between members of different species [19] – are specific types of symbiotic relationships found in nature that result in cross-species interactions. By formalizing such principles in robotics, one can establish symbiotic (i.e., collaborative) arrangements between robots with different functionalities. These types of interactions are especially suitable for the setting of long-duration autonomy, where there is a tight coupling between an organism (robot) and its environment (workspace) [20], [21].

One way to capture the capabilities of a heterogeneous multi-robot team is through control barrier functions (CBFs), which provide a means to guarantee that dynamical systems remain within a safe set [22]. In particular, CBFs can be utilized to capture the individual robots' respective capabilities and then, through collaborative arrangements, be made to expand, resulting in new functionalities that did not manifest themselves individually [23].

It is worth mentioning that CBFs, used as a mechanism to encode the safe operating regions (or safe set) of robots in engineered systems, is similar to the ecological concept of a niche [24] – the match of an organism to a set of environmental conditions where it can survive and produce offspring successfully. By collaborating, robots can acquire new capabilities, i.e., expand their safe set, which is similar to the ecological concept of niche expansion [25] – the presence of another species has expanded the area in which an organism can survive and produce offspring successfully.

This paper builds, and significantly expands, upon the findings presented in [23], where the idea of encoding pairwise collaborations in multi-robot systems with barrier functions and a hybrid automata-based collaboration framework was introduced. The novel contributions of this paper are four-fold:

- Consider pairwise collaborations between $N > 2$ robots;
- Formalize a pairwise barrier function – based on state-dependent restrictions and safe regions of operation – to capture the influence between robots during collaborative interactions;
- Extend the notation of safe set (dependent on the domain) to safely reachable set (dependent on the domain, initial states, and dynamics), allowing us to establish the conditions under which pairwise collaboration is beneficial, together with introducing the notion of symbiosis into heterogeneous multi-robot teams;
- Include extensive simulation and experimental results to demonstrate the efficacy of the pairwise collaboration framework.

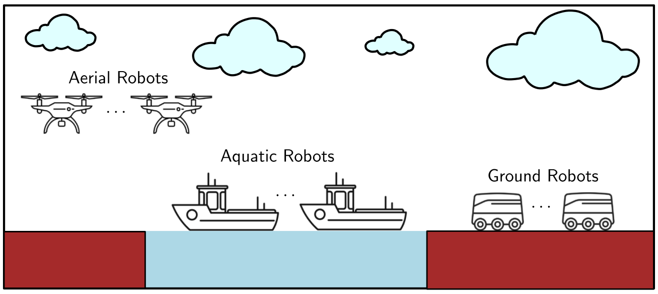


FIGURE 1. Example scenario: N robots that are placed into $M = 3$ capability types (ground, aquatic, and aerial robots).

The remainder of this paper is organized as follows: Section II introduces the problem setting for pairwise collaborations in heterogeneous multi-robot teams. Section III provides relevant background material on encoding pairwise collaborations through barrier functions, along with providing details on "Why should robots collaborate?", "When should robots collaborate?", and "How can robots collaborate?". Section IV validates our findings with case studies performed on a team of mobile robots. Section V contains concluding remarks.

II. PROBLEM SETTING

Consider a scenario where N robots can be placed into M categories based on their capability types, e.g., as observed in Fig. 1, where each robot is associated with a state vector, $x_i \in \mathcal{X}_i \subset \mathbb{R}^{n_{x_i}}$, and a control input vector, $u_i \in \mathcal{U}_i \subset \mathbb{R}^{n_{u_i}}$, $\forall i \in \mathcal{N} = \{1, \ldots, N\}$ (index set of robots). In addition, let $x = [x_1^\mathsf{T}, \ldots, x_N^\mathsf{T}]^\mathsf{T} \in \mathcal{X} = \mathcal{X}_1 \times \ldots \times \mathcal{X}_N$ and $u = [u_1^\mathsf{T}, \ldots, u_N^\mathsf{T}]^\mathsf{T} \in \mathcal{U} = \mathcal{U}_1 \times \ldots \times \mathcal{U}_N$ denote the stacked state and control input vectors, respectively.

We assume that each robot is tasked with achieving some final state that can either be realized by the robot itself or by collaborating with other individuals. When robots are engaged in collaboration, we further assume that the interactions are pairwise – i.e., one robot provides help while another receives help – and through collaboration, the help-receiving robot can expand its capabilities by exploiting the diverse functionalities of the help-providing robot. For example, an aquatic robot can ferry a ground robot in water-based terrains, or a ground robot can carry an aquatic robot on land-based terrains.

In [23], a pairwise collaboration framework was introduced that enables robots to provide and receive help when functioning in the high-level operating modes: 'Individual Tasks' (mode q_1; robots operate independently to achieve their desired final state), 'Collaboration Setup' (mode q_2; robots i and j prepare themselves to collaborate), and 'Collaborative Act' (mode q_3; robots i and j engage in pairwise collaboration), where robot i denotes the help-receiving robot and robot j denotes the help-providing robot. By employing this framework, the individual robots can operate in new parts of the state space that were not accessible without help, i.e., each robot can expand its set of achievable final states by collaborating.

While robots can acquire new capabilities by collaborating, they can also potentially incur restrictions on their dynamics, states, and control inputs. For example, pairwise collaboration can impose restrictions on the coordination of movement and orientation or require the individual robots to move together, which may even result in losing some degree of control authority as the cost to collaborate. However, the specifics of such restrictions are scenario-dependent, as they are a function of the individual robots' collaborative arrangements and underlying dynamics.

Suppose the pairwise collaboration restrictions between a robot i (help-receiving) and a robot j (help-providing) are captured by

$$\psi_{ij}(x_i, x_j) = 0, \tag{1}$$

$$\phi_{ij}(x_i, x_j) \leq 0, \tag{2}$$

where $\psi_{ij}(x_i, x_j) = [\psi_{ij,1}(x_i, x_j), \ldots, \psi_{ij,n_{\psi_{ij}}}(x_i, x_j)]^\mathsf{T}$ and $\phi_{ij}(x_i, x_j) = [\phi_{ij,1}(x_i, x_j), \ldots, \phi_{ij,n_{\phi_{ij}}}(x_i, x_j)]^\mathsf{T}$ are the state-dependent equality and inequality restrictions, respectively, for the particular collaborative arrangement between robots i and j, with $|\psi_{ij}(x_i, x_j)| = n_{\psi_{ij}}$ and $|\phi_{ij}(x_i, x_j)| = n_{\phi_{ij}}$ ($|\cdot|$ denotes the cardinality). Note that robots i and j must incur restrictions to engage in pairwise collaboration, so we assume it is infeasible for them to work together if $n_{\psi_{ij}} = n_{\phi_{ij}} = 0$.

The robots' dynamics are modeled as switched systems [26], which exhibit state-dependent switching between the high-level operating modes of the pairwise collaboration framework $\mathcal{Q} = \{q_1, q_2, q_3\}$, with a control-affine form, given by

$$\dot{x}_i = f_{i,\sigma_i(x)}(x_i) + g_{i,\sigma_i(x)}(x_i)u_i, \tag{3}$$

where $\sigma_i : \mathcal{X} \to \mathcal{Q}$ is a piecewise constant signal that is continuous from the right everywhere, i.e., $\sigma_i(x(t)) = \lim_{\tau\to t^+} \sigma_i(x(\tau))\ \forall\tau \geq 0$. In addition, $f_{i,\sigma_i(x)} : \mathbb{R}^{n_{x_i}} \to \mathbb{R}^{n_{x_i}}$ and $g_{i,\sigma_i(x)} : \mathbb{R}^{n_{x_i}} \to \mathbb{R}^{n_{x_i}\times n_{u_i}}$ are locally Lipschitz vector fields within each operating mode $\sigma_i(x) \in \mathcal{Q}$. However, given the nature of pairwise collaborations, the dynamics of robot i can potentially change, i.e.,

$$f_{i,q_1}(x_i) = f_{i,q_2}(x_i) \neq f_{i,q_3}(x_i), \tag{4}$$

$$g_{i,q_1}(x_i) = g_{i,q_2}(x_i) \neq g_{i,q_3}(x_i), \tag{5}$$

where $\sigma_i(x) \in \{q_1, q_2\}$ indicates that robot i is not actively engaged in pairwise collaboration, while $\sigma_i(x) \in \{q_3\}$ indicates that robot i is; e.g., the dynamics of an aerial robot and a ground robot operating independently can have a different structure than the dynamics of an aerial robot lifting a ground robot in the air.

Notably, it is possible for the individual robots' control law to change based on the switching signal whose details – i.e., state-dependent conditions for each operating mode – are discussed in Section III-D.

III. PAIRWISE COLLABORATIONS

This section presents background on how barrier functions can be utilized for pairwise collaborations in a heterogeneous multi-robot team, as was done in [23], along with details on "Why?", "When?", and "How?" robots should collaborate.

A. BACKGROUND

When viewed in isolation, robot i can be associated with safe regions of operation defined by the zero-superlevel set, $\mathcal{S}_i$, of a continuously differentiable function $h_i : \mathcal{X}_i \to \mathbb{R}$, where h_i is a control barrier function if there exists an extended class $\mathcal{K}_\infty$ function, $\alpha(\cdot)$, such that

$$\dot{h}_i(x_i, u_i) \geq -\alpha(h_i(x_i)), \tag{6}$$

is satisfied for all time [22]. Then, $\mathcal{S}_i$ can be rendered forward invariant, with respect to (3), as long as robot i's initial state belongs to its safe set, i.e., $x_i(t_0) \in \mathcal{S}_i$.

In this paper, barrier functions are used to capture the safe operating regions of each robot functioning by itself, i.e., in the absence of other robots. Specifically, robot i's safe set, $\mathcal{S}_i$, can be represented as

$$\mathcal{S}_i = \{x_i \in \mathcal{X}_i \mid h_i(x_i) \geq 0\} \subseteq \mathcal{X}_i, \tag{7}$$

which we interpret as the regions that a robot can operate in effectively and safely by itself, akin to the ecological concept of a niche [24].

In the presence of others, individual robots have the potential to influence each other through pairwise interactions, which can be captured with pairwise barrier functions. For instance, robots typically cannot coexist at the same location and time, as this would lead to collisions, implying that the safe set can potentially shrink for robots considering collision avoidance safety constraints, as observed in [27], [28], [29].

Alternatively, a robot's safe operating regions can potentially expand due to the presence of other robots, akin to the ecological concept of niche expansion [25]. Hence, the pairwise influence that robot j at state x_j has on robot i at state x_i during collaboration can be captured through the pairwise barrier function $h_{ij}(x_i, x_j)$, as

$$h_{ij}(x_i, x_j) = \begin{cases} \Gamma(x_i, x_j), & \begin{array}{l} \text{if } \psi_{ij}(x_i, x_j) = 0 \wedge \\ \phi_{ij}(x_i, x_j) \leq 0 \wedge \\ h_i(x_i) < 0 \end{array} \\ 0, & \text{otherwise} \end{cases}, \tag{8}$$

where $\Gamma : \mathcal{X}_i \times \mathcal{X}_j \to \mathbb{R}_{>0}$ is a positive-valued scalar function, and we adopt the notational convention that $h_{ii}(x_i, x_i) = 0$. Thus, $h_{ij}(x_i, x_j)$ must be sufficiently positive – when (1) and (2) hold and $x_i \notin \mathcal{S}_i$ – for collaboration between robots i and j to be helpful in the states x_i and x_j.

We can now combine the individual and pairwise barrier functions, such that robot i at x_i can be made safe by robot j at x_j, as

$$H_{ij}(x_i, x_j) = h_i(x_i) + h_{ij}(x_i, x_j), \tag{9}$$

as done in [23].

The pairwise safe set for robot i, which includes the regions that can be rendered safe by the help of a particular robot j, is defined as

$$\mathcal{S}_{ij} = \{x_i \in \mathcal{X}_i \mid \exists x_j \in \mathcal{X}_j \; s.t. \; H_{ij}(x_i, x_j) \geq 0\} \subseteq \mathcal{X}_i. \quad (10)$$

Here, the existential quantifier, $\exists$, encodes that if a particular robot j is in state x_j, the corresponding state x_i for robot i would be rendered safe thanks to robot j's help. The interpretation is that a region can be made safe when robot i and a particular robot j, i.e., a potential collaborator, form a suitable collaborative partnership.

Any robot j at state x_j is identified as a potential collaborator to robot i at state x_i if its index is contained in

$$\mathcal{F}_i(x) = \{j \in \mathcal{N} \mid H_{ij}(x_i, x_j) \geq 0\}, \quad (11)$$

where $\mathcal{F}_i(x) \subseteq \mathcal{N}$ is defined as the set of potential collaborators to robot i at state x_i. It should be mentioned that the set of potential collaborators always contains the index of robot i. For example, if robot i is not collaborating with other robots, then $H_{ii}(x_i, x_i) = h_i(x_i)$, implying that $\mathcal{F}_i(x) = \{i\}$ (singleton set).

The total safe set for robot i, which includes all pairwise safe sets, is then defined as

$$\bar{\mathcal{S}}_i = \bigcup_{j \in \mathcal{N}} \mathcal{S}_{ij}, \quad (12)$$

where we adopt the notational convention that $\mathcal{S}_{ii} = \mathcal{S}_i$.

The next step is to encode the individual robots' effective regions of operation while considering their respective dynamics and initial states. In other words, we are interested in defining a reachable set [30] – the set of states attainable from any trajectory that begins at some initial state – which guarantees safety.

When establishing a reachable set that respects safety constraints, we assume that the individual robots' can generate and implement a suitable control signal that guarantees each robot's safety, whether they collaborate or not, while progressing toward their desired final state. Thus, if robot i (help-receiving) can work with robot j (help-providing), identified as a potential collaborator, then it is assumed that an appropriately designed control signal is applied by robot i such that its safe operating region can expand through pairwise collaboration.

We define the individual safely reachable set – i.e., regions of the state space that robot i can reach safely without help from some initial condition – as

$$\begin{aligned} &\mathcal{R}_i(x_{i,0}) = \\ &\{\bar{x}_i \in \mathcal{X}_i \mid \exists \bar{t} \geq t_0 \text{ and } \exists u_i(t) \in \mathcal{U}_{i[t_0,\bar{t}]} \; s.t. \; \forall t \in [t_0, \bar{t}] \\ &\qquad x_i(t) \text{ satisfies (3) from } x_i(t_0) = x_{i,0} \text{ under} \\ &\qquad u_i(t) \wedge \sigma_i(x(t)) = q_1, \\ &\qquad \dot{h}_i(x_i(t), u_i(t)) \geq -\alpha_i(h_i(x_i(t))), \\ &\qquad \text{and } x_i(\bar{t}) = \bar{x}_i\} \end{aligned} \quad (13)$$

where $x_i(t_0) \in \mathcal{S}_i$ and $x_i(\bar{t}) \in \mathcal{S}_i$ are robot i's initial and final state, respectively; $u_i(t) \in \mathcal{U}_{i[t_0,\bar{t}]}$ is robot i's control trajectory, and $\alpha_i(\cdot)$ is robot i's extended class $\mathcal{K}_\infty$ function.

However, there may be cases in which robot i requires assistance from other robots to achieve a desired final state in the presence of unsafe regions of operation. For instance, a landscape composed of multiple terrains created by natural disruptions, e.g., wildfire, earthquake, or flooding, may necessitate that robot i receives help from several robots j, identified as potential collaborators, rather than a single robot j, to traverse the multi-terrain landscape. In such a setting, if pairwise collaboration between multiple robots is beneficial – i.e., expansion of safe operating region – and possible – i.e., the other robots j, identified as potential collaborators, can reach robot i – then it is assumed that the individual robots can utilize appropriately designed control signals such that robot i can successfully achieve a final state with the help of other robots.

We define the total safely reachable set – i.e., regions of the state space that robot i can reach safely both with or without help from other robots j, identified as potential collaborators, from some initial conditions – as

$$\begin{aligned} &\bar{\mathcal{R}}_i(x_0) = \\ &\{\bar{x}_i \in \mathcal{X}_i \mid \exists \bar{t} \geq t_0 \text{ and } \exists u(t) \in \mathcal{U}_{[t_0,\bar{t}]} \; s.t. \; \forall t \in [t_0, \bar{t}] \\ &\qquad x_\ell(t) \text{ satisfies (3) from } x_\ell(t_0) = x_{\ell,0} \text{ under} \\ &\qquad u_\ell(t) \, \forall \ell \in \mathcal{N}, \\ &\qquad \exists j(t) \in \mathcal{F}_i(x(t)) \; s.t. \\ &\qquad \dot{H}_{ij(t)}(x_i(t), x_{j(t)}(t), u_i(t), u_{j(t)}(t)) \geq \\ &\qquad\qquad - \alpha_i(H_{ij(t)}(x_i(t), x_{j(t)}(t))) \wedge \\ &\qquad \dot{H}_{j(t)i}(x_{j(t)}(t), x_i(t), u_{j(t)}(t), u_i(t)) \geq \\ &\qquad\qquad - \alpha_{j(t)}(H_{j(t)i}(x_{j(t)}(t), x_i(t))), \\ &\qquad \text{and } x_i(\bar{t}) = \bar{x}_i\} \end{aligned} \quad (14)$$

where $[x_1(t_0)^\mathsf{T}, \ldots, x_N(t_0)^\mathsf{T}]^\mathsf{T} = x_0 \in \mathcal{S}$ are the robots' initial states; $x_i(\bar{t}) \in \bar{\mathcal{S}}_i$ is robot i's final state; $u(t) \in \mathcal{U}_{[t_0,\bar{t}]}$ are the robots' control trajectories; and $\alpha_i(\cdot)$ and $\alpha_{j(t)}(\cdot)$ are the extended class $\mathcal{K}_\infty$ function of robots i and $j(t) \in \mathcal{F}_i(x)$, respectively.

It is worth mentioning that, by the construction of (8), $\mathcal{R}_i(x_{i,0}) \subseteq \bar{\mathcal{R}}_i(x_0)$, implying that collaborative arrangements cannot contract the individual robots' safe operating regions, only expand them.

Next, we investigate the questions of "Why should robots collaborate?", "When should robots collaborate?", and "How can such pairwise collaborations be achieved?".

B. WHY SHOULD ROBOTS COLLABORATE?

Pairwise collaboration is beneficial when robot i's individual safely reachable set, given by (13), expands, meaning that robot i has access to new parts of the state space it previously did not alone.

With the individual safely reachable set, $\mathcal{R}_i(x_{i,0})$, and the total safe set, $\bar{\mathcal{S}}_i$, in mind, we can establish upper- and lower-bounds, using set comparisons, on the total safely reachable set, given as

$$\mathcal{R}_i(x_{i,0}) \subseteq \bar{\mathcal{R}}_i(x_0) \subseteq \bar{\mathcal{S}}_i. \tag{15}$$

In (15), the lower-bound is the individual safely reachable set, $\mathcal{R}_i(x_{i,0})$, since, in the worst-case scenario, no safe operating region expansion is attainable, we have $\mathcal{R}_i(x_{i,0}) = \bar{\mathcal{R}}_i(x_0)$; whereas the upper-bound is the total safe set, $\bar{\mathcal{S}}_i$, since, in the best-case scenario, maximum safe operating region expansion is attainable, we have $\bar{\mathcal{S}}_i = \bar{\mathcal{R}}_i(x_0)$.

Before robot i (help-receiving) collaborates with other robots j (help-providing), identified as potential collaborators, it should determine if forming collaborative arrangements would be beneficial.

A necessary condition for pairwise collaboration to be beneficial is

$$\mathcal{S}_i \subset \bar{\mathcal{S}}_i, \tag{16}$$

i.e., $\exists j \in \mathcal{F}_i(x)$ such that $h_{ij}(x_i, x_j)$ can possibly be made positive for pairwise collaboration between robots i and j to be helpful in the states x_i and x_j.

A necessary and sufficient condition for pairwise collaboration to be beneficial

$$\mathcal{R}_i(x_{i,0}) \subset \bar{\mathcal{R}}_i(x_0), \tag{17}$$

as it takes the robots' initial states, dynamics, and applicable control laws into account; i.e., $\exists u(t) \in \mathcal{U}_{[t_0,\bar{t}]}$ such that $h_{ij}(x_i, x_j)$ is made positive for pairwise collaboration between robots i and j to be helpful in the states x_i and x_j by expanding the reach of robot i.

It is now possible to draw a parallel to the ecological concept of symbiosis [31] – an interaction between two dissimilar species living in close physical association; a concept transferable to engineered systems. One interpretation of symbiosis in engineered systems could be termed "robot mutualism" – a jointly beneficial interaction between robots with different functionalities – whereas another could be termed "robot commensalism" – a unilaterally beneficial interaction between robots with different functionalities.

A robot mutualism occurs when there exists a particular collaborative arrangement between robots i and j that results in the expansion of each robot's individual safely reachable set, i.e.,

$$\mathcal{R}_i(x_{i,0}) \subset \bar{\mathcal{R}}_i(x_0) \wedge \mathcal{R}_j(x_{j,0}) \subset \bar{\mathcal{R}}_j(x_0), \tag{18}$$

where both robots benefit from collaborating with each other. Note that the expansion of the robots' individual safely reachable sets does not have to, although it can, happen simultaneously.

A robot commensalism occurs when there exists a particular collaborative arrangement between a robot i (help-receiving) and a robot j (help-providing) that results in the expansion of robot i's individual safely reachable set, i.e.,

$$\mathcal{R}_i(x_{i,0}) \subset \bar{\mathcal{R}}_i(x_0) \wedge \mathcal{R}_j(x_{j,0}) = \bar{\mathcal{R}}_j(x_0), \tag{19}$$

where robot i benefits from the help provided by robot j and robot j derives neither benefit nor cost from collaborating with robot i.

C. WHEN SHOULD ROBOTS COLLABORATE?

For robot i (help-receiving) and robot j (help-providing), certain conditions must be satisfied before collaborating, i.e., mode q_2, and then while collaborating, i.e., mode q_3. Moreover, any robot j, identified as a potential collaborator, must be available and willing to offer its services to any robot i that requests assistance.

However, before robot i requests assistance from a robot $j \in \mathcal{F}_i(x)$, it must check if it would enter an unsafe region while operating alone. Since each robot is associated with a safe operating region, given by (7), there could be areas in which a robot cannot reach by itself; i.e., the robots are limited to functioning in their individual safely reachable set, given by (13), without receiving any help.

To check this, we must first consider the nominal dynamics of robot i, i.e., the behavior robot i would exhibit when functioning in mode q_1 without considering any safety constraints, given by

$$\dot{x}_{i,\text{nom}} = f_{i,q_1}(x_i) + g_{i,q_1}(x_i)\hat{u}_i, \tag{20}$$

where $\hat{u}_i$ is the nominal controller, defined as

$$\hat{u}_i = \begin{cases} \hat{u}_{i,q_1}, & \text{if } \sigma_i(x) = q_1 \\ \hat{u}_{i,q_2}, & \text{if } \sigma_i(x) = q_2 \\ \hat{u}_{i,q_3}, & \text{if } \sigma_i(x) = q_3 \end{cases}. \tag{21}$$

where $\hat{u}_{i,q_1}$, $\hat{u}_{i,q_2}$, and $\hat{u}_{i,q_3}$ is the nominal controller for mode q_1, q_2, and q_3, respectively.

For each mode, the individual robots, whether they are help-providing or help-receiving, are assumed to have a nominal controller, $\hat{u}_i$, whose design depends on the task at hand, to have them progress towards an objective while ignoring safety constraints, which is different from the control input, u_i, applied in (13) and (14); since u_i guarantees safety over a trajectory, whereas $\hat{u}_i$ does not. Of course, $\hat{u}_{i,q_2}$ for a help-receiving robot may differ from $\hat{u}_{j,q_2}$ for a help-providing robot, as robots i and j could require different control laws to appropriately set themselves up to collaborate.

We can now establish the collaboration signal condition, which enables robot i to check whether it would enter an unsafe region, i.e., $h_i(x_i) < 0$, when following its nominal dynamics, $\dot{x}_{i,\text{nom}}$, while on the boundary of its safe operating region, $x_i \in \partial\mathcal{S}_i$, given by

$$\langle \nabla h_i(x_i), \dot{x}_{i,\text{nom}} \rangle < 0 \wedge h_i(x_i) = 0, \tag{22}$$

allowing robot i to determine if it requires assistance from any robot j, identified as a potential collaborator.

To begin collaborating, a particular robot i and a particular robot j, identified as a potential collaborator, must be path-connected, i.e., the intersection of their individual safely reachable sets is non-empty, i.e., $\mathcal{R}_i(x_{i,0}) \cap \mathcal{R}_j(x_{j,0}) \neq \emptyset$. The robots also must be able to reach the "collaboration submanifold" – a topological space constructed from the state-dependent restrictions, given by (1) and (2), which depend on the particular collaborative arrangement between two robots – by admitting a state that is contained within their respective individual safely reachable set. This enables the state-dependent restrictions to be met, together with ensuring the robots' barrier functions satisfy (22) and they remain safe while progressing to the collaboration submanifold in mode q_2.

Once the state-dependent restrictions are met, robots i and j must continue to maintain them throughout the entire duration of their collaborative arrangement, i.e., $\psi_{ij}(x_i(t), x_j(t)) = 0$ and $\phi_{ij}(x_i(t), x_j(t)) \leq 0 \; \forall t \in [t_1, t_2]$, where t_1 is the time at which collaboration begins – i.e., time of transition from mode q_2 to q_3 – and t_2 is the time at which collaboration ends – i.e., time of transition from mode q_3 to q_1. Moreover, the collaborative barrier function, given by (9), must be non-negative for both robots i and j such that they remain safe during pairwise collaboration, while evolving on the collaboration submanifold in mode q_3. Then, pairwise collaboration ends when the state-dependent restrictions are no longer needed, and the individual robots can safely progress towards their tasks alone.

D. HOW CAN ROBOTS COLLABORATE?

In the pairwise collaboration framework, it is assumed that robot i (help-receiving) can broadcast its barrier function, $h_i(x_i)$, to other robots j (help-providing), identified as potential collaborators, to request assistance. Once a suitable pairing between robot i and robot j is determined, it is also assumed that the robots can share their respective state information within modes q_2 and q_3.

As discussed in Section II, the pairwise collaboration framework has three high-level operating modes: 'Individual Tasks' (mode q_1), 'Collaboration Setup' (mode q_2), and 'Collaborative Act' (mode q_3), which, for a particular robot i (help-receiving) and a particular robot j (help-providing), identified as a potential collaborator, can be described as:

- Mode q_1: Robots i and j progress towards their respective final states by themselves;
- Mode q_2: Robots i and j coordinate their behaviors to meet the state-dependent restrictions, i.e., (1) and (2);
- Mode q_3: Robot i receives help from robot j such that robot i is rendered pairwise safe, as it would be unsafe by itself, i.e., $H_{ij}(x_i, x_j) \geq 0$ and $h_i(x_i) < 0$, while maintaining the state-dependent restrictions.

For the particular collaborative arrangement between robots i and j, the modes can be formalized in terms of barrier functions as

$$\underset{q_1}{\text{Mode}}: \begin{cases} h_i(x_i) \geq 0 \\ h_j(x_j) \geq 0 \end{cases}, \tag{23}$$

$$\underset{q_2}{\text{Mode}}: \begin{cases} \langle \nabla h_i(x_i), \dot{x}_{i,\text{nom}} \rangle < 0 \wedge h_i(x_i) = 0 \\ h_j(x_j) \geq 0 \end{cases}, \tag{24}$$

$$\underset{q_3}{\text{Mode}}: \begin{cases} h_i(x_i) < 0 \wedge h_{ij}(x_i, x_j) > 0 \wedge H_{ij}(x_i, x_j) \geq 0 \\ h_j(x_j) \geq 0 \end{cases}, \tag{25}$$

where there will be a transition from mode q_1 to q_2, i.e., from 'Individual Tasks' to 'Collaboration Setup' when the conditions in (24) hold; there will be a transition from mode q_2 to q_3, i.e., from 'Collaboration Setup' to 'Collaborative Act' when the conditions in (25) hold; and there will be a transition from mode q_3 to q_1, i.e., from 'Collaborative Act' to 'Individual Tasks' when the conditions in (23) hold.

In addition, we can express the switching signal between a particular robot i (help-receiving) and a particular robot j (help-providing), identified as a potential collaborator, in terms of the barrier function conditions as

$$\sigma_i(x) = \sigma_j(x) = \begin{cases} q_1, & \text{if (23) is satisfied} \\ q_2, & \text{if (24) is satisfied} \\ q_3, & \text{if (25) is satisfied} \end{cases}. \tag{26}$$

The barrier function conditions, given by (23)–(25), and the switching signal, given by (26), are immediately transferable to pairwise collaborations between $N > 2$ robots. However, in that case, we must arrange suitable pairings between help-providing and help-receiving robots before they engage in collaboration, which is realizable through the use of matching algorithms [32], [33], [34]; help-receiving robots are assigned to help-providing robots such that constraints, e.g., safe operating regions and selection preferences, are satisfied. A popular method that solves the assignment problem in polynomial time is the Hungarian algorithm [35], where the run-time complexity is $\mathcal{O}(n^3)$ and space complexity is $\mathcal{O}(n^2)$ with n being the total number of help-receiving and help-providing robots.

Within the pairwise collaboration framework, we implement an optimization-based control strategy that enforces safety constraints both with and without collaboration. To guarantee robot i remains safe for all time, with or without the help of other robots, we impose the safety certificate

$$\dot{H}_{ij}(x, u) \geq -\alpha_i(H_{ij}(x)), \tag{27}$$

where $\dot{H}_{ij}(x, u) = L_{f_{\sigma(x)}} H_{ij}(x) + L_{g_{\sigma(x)}} H_{ij}(x) u$ such that $L_{f_{\sigma(x)}} H_{ij}(x) = \nabla H_{ij}(x) \cdot f_{\sigma(x)}(x)$ and $L_{g_{\sigma(x)}} H_{ij}(x) = \nabla H_{ij}(x) \cdot g_{\sigma(x)}(x)$, with the vector fields $f_{\sigma(x)}(x) = [f_{1,\sigma_1(x)}(x_1)^{\mathsf{T}}, \ldots, f_{N,\sigma_N(x)}(x_N)^{\mathsf{T}}]^{\mathsf{T}}$ and $g_{\sigma(x)}(x) = [g_{1,\sigma_1(x)}(x_1)^{\mathsf{T}}, \ldots, g_{N,\sigma_N(x)}(x_N)^{\mathsf{T}}]^{\mathsf{T}}$, using Lie derivative notation.

A safety-critical controller that guarantees the individual robots remain within their respective safe sets while attempting to track their nominal controller, $\hat{u}_i$, as closely as possible, at each point in time, can be formulated as a Quadratic Program (QP) [36], where we denote the actual and nominal control inputs as u_i (decision variable) and $\hat{u}_i$, respectively,

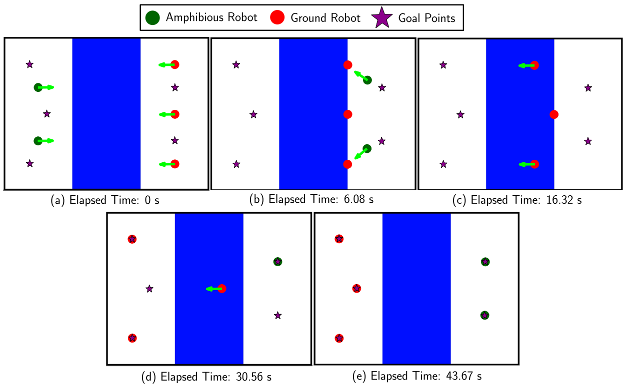


FIGURE 2. Scenario 1: Robot commensalism between three ground robots (red) and two amphibious robots (green) tasked with safely reaching their respective goal points (purple stars), within an environment comprised of land (white pixels) and water (blue pixels) terrains. [Supplemental Video: https://youtu.be/Q5G8K3QuOSo].

given by

$$u^* = \underset{u \in \mathcal{U}}{\operatorname{argmin}} \quad \frac{1}{2} \sum_{i=1}^{N} \|u_i - \hat{u}_i\|_2^2$$
$$\text{s.t.} \quad A(x)u \le b(x), \tag{28}$$

where $A(x)$ and $b(x)$ are linear (in the decision variable u) constraints. For example, $-L_{g_{\sigma(x)}} H_{ij}(x)u \le \alpha(H_{ij}(x)) + L_{f_{\sigma(x)}} H_{ij}(x)$ is the a linear constraint enabling pairwise collaborative interactions to safely occur between relevant robots. In addition, $A(x)$ and $b(x)$ could also be comprised of other barrier function constraints or actuator limitations.

In practice, (28) is solved as a finite-dimensional convex problem – even though the control input, u_i, is continuous – as there is a conversion from analog-to-digital during implementation.

IV. CASE STUDIES

Two case studies are considered in simulations and hardware experiments. Snapshots of the robots' locations and the minimum value of each robot's pairwise barrier functions for collaboration, defined as $H_i^{\min}(x) = \min_{j \in \mathcal{N}} H_{ij}(x_i, x_j)$, are provided.

Note that we do not explicitly compute the safely reachable sets in the case studies, as their expansion is apparent by inspection. More complex scenarios, however, would require additional analysis, which is the subject of current research.

A. SIMULATIONS

Simulations are conducted for two scenarios involving disruptions to the environment. The first scenario – ground and amphibious robots (illustrated in Fig. 2) – considers an event that causes the landscape to be comprised of water terrain ($\mathcal{D}_{\text{water}}$; blue pixels) and land terrain ($\mathcal{D}_{\text{land}}$; white pixels), e.g., through flooding. The second example scenario – triangle and square robots (illustrated in Fig. 3) – considers a landscape that contains ledges, e.g., through landslides.

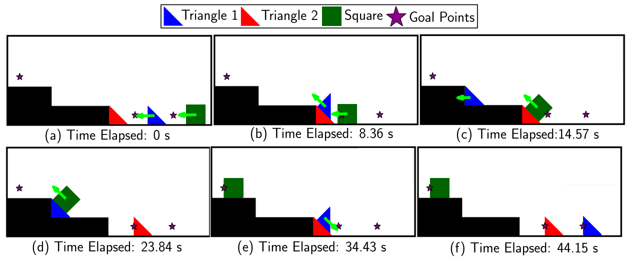


FIGURE 3. Scenario 2: Robot mutualism between two triangle-shaped robots (blue and red) and one square-shaped robot (green) in a landscape that contains ledges (black pixels). [Supplemental Video: https://youtu.be/w1sgrPFxFvM].

1) SCENARIO 1: GROUND-AMPHIBIOUS ROBOTS

Each robot's states and control inputs are defined as $x_i = [p_{i,x}, p_{i,y}, v_{i,x}, v_{i,y}]^\mathsf{T} \in \mathcal{X}_i \subset \mathbb{R}^4$ and $u_i \in \mathcal{U}_i \subset \mathbb{R}^2$, respectively, where $p_{i,x}, p_{i,y}$ represent the planar position states and $v_{i,x}, v_{i,y}$ represent the planar velocity states. It is also assumed that each robot – modeled as a point mass – exhibits double integrator dynamics, i.e., $\ddot{x}_i = u_i$.

We let $\alpha(r) = r$; sampling period be $T = 0.01$ s; water terrain "strip" location, with respect to the x-coordinate, be $[-\ell/2, \ell/2]$ m where $\ell = 2$ m; and the boundaries of the workspace be given by $[-x_{\text{min-max}}, x_{\text{min-max}}]$ m by $[-y_{\text{min-max}}, y_{\text{min-max}}]$ m in the x- and y-coordinates, respectively, where $x_{\text{min-max}} = 3$ m and $y_{\text{min-max}} = 1$ m.

The ground robots' individual barrier function, $h_g(x_g)$, ensures safe operation on land-based terrains only, whereas the amphibious robots' individual barrier function, $h_a(x_a)$, ensures safe operation on both land-based and water-based terrains. Since the robots employ double integrator dynamics, the barrier function constraint must be designed for a system with relative degree two [37].

The ground robots' individual and pairwise barrier functions are given by

$$h_g(x_g) = 2p_{g,x}v_{g,x} + \kappa(p_{g,x}^2 - (\ell/2)^2), \tag{29}$$

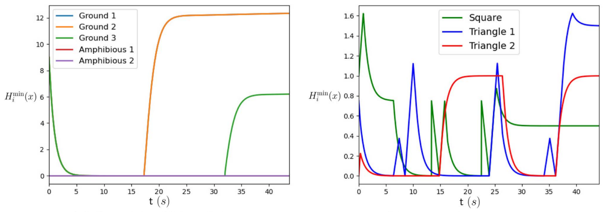
$$h_{ga}(x_g, x_a) = \begin{cases} -h_g(x_g), & \text{if } \psi_{ga}(x_g, x_a) = 0 \wedge \\ & \quad h_g(x_g) < 0 \\ 0, & \text{else} \end{cases}, \tag{30}$$

respectively, where $\psi_{ga}(x_g, x_a) = \|p_g - p_a\|_2$ is the ground robots' state-dependent restriction required for pairwise collaboration with the amphibious robots, interpreted as the amphibious robot carrying the ground robot, and κ is a class-$\mathcal{K}$ function, assumed here to be linear with a positive coefficient, for the higher order barrier function.

The amphibious robots' individual and pairwise barrier functions are given by

$$h_a(x_a) = h_{ag}(x_a, x_g) = 0, \tag{31}$$

which are both zero as the amphibious robots can operate safely over land-based and water-based terrains and the ground robots are assumed to have no pairwise influence on the amphibious robots for collaboration purposes.



(a) Ground-Amphibious Robots (a) Triangle-Square Robots

FIGURE 4. Minimum value of each robot's pairwise barrier functions for collaboration: (a) ground-amphibious robots and (b) triangle-square robots.

There is only an expansion of the ground robots' individual safely reachable set from the amphibious robots' help – i.e., $\mathcal{R}_g(x_{g,0}) \subset \bar{\mathcal{R}}_g(x_0)$ while $\mathcal{R}_a(x_{a,0}) = \bar{\mathcal{R}}_a(x_0)$, which happens when the amphibious robots ferry the ground robots across water-based terrains such that the entire domain becomes (safely) reachable to the ground robots, implying a robot commensalism – as only some individuals experience a net benefit from participating in the collaborative interactions.

This scenario also considers amphibious robots that must choose who to help between the help-requesting ground robots. One approach is to execute a task assignment algorithm, e.g., the Hungarian algorithm [35]. Alternatively, selection can be made based on maximizing a desired performance metric, which is the approach adopted here, where the amphibious robots made their selections based on maximizing battery life (or minimizing energy expenditure) by pairing with the nearest help-receiving robot.

We provide snapshots of the simulation in Fig. 2(a)–(e). Fig. 2(a) shows the initial locations of the ground robots (red) and amphibious robots (green). Fig. 2(b) shows the two amphibious robots moving toward their selected ground robots on the boundary. Fig. 2(c) shows the two amphibious robots ferrying the ground robots across the water terrain. Fig. 2(d) shows an amphibious robot ferrying a ground robot while other robots progress safely toward their respective goal points. Fig. 2(e) shows the final locations of the ground and amphibious robots. Fig. 4(a) portrays the minimum value of each robot's pairwise barrier functions for collaboration, highlighting that each robot remained safe throughout the entire simulation, i.e., $H_i^{\min}(x) \geq 0 \; \forall t$, both with or without collaboration.

2) SCENARIO 2: TRIANGLE-SQUARE ROBOTS

Each robot's states and control inputs are defined as $x_i = [p_{i,x}, p_{i,y}]^{\mathsf{T}} \in \mathcal{X}_i \subset \mathbb{R}^2$ and $u_i \in \mathcal{U}_i \subset \mathbb{R}^2$, respectively, where $p_{i,x}$, $p_{i,y}$ represent the planar position states. In addition, Square has a length of ℓ_s, whereas Triangle 1 and Triangle 2 both have a base length of ℓ_t and height of h. It is also assumed that each robot – modeled as a convex polytope; defined as $\mathcal{P}_s$ (Square), $\mathcal{P}_{t_1}$ (Triangle 1), and $\mathcal{P}_{t_2}$ (Triangle 2) – exhibits single integrator dynamics, i.e., $\dot{x}_i = u_i$. Each robot has a single actuator that generates motion in the direction parallel to the bottom edge of each robot.

We let $\alpha(r) = 100r$; sampling period be $T = 0.01$ s; first ledge's location, with respect to the x-coordinate, be 0 m (the origin); second ledge's location, with respect to the x-coordinate, be $-3\ell_t$; and the workspace boundaries be given by $[-\ell, \ell]$ m by $[0, 3h]$ m in the x- and y-coordinates, respectively, where $\ell = 4$ m. We also assume that the triangle-shaped robots' height and base length and the square-shaped robot's length are $h = \ell_t = \ell_s = 0.75$ m and that the height of the first and second ledges are h and $2h$, respectively, to ensure a smooth transition.

Triangle 1's individual barrier function, $h_{t_1}(x_{t_1})$, ensures safe operation anywhere outside a distance ℓ_t from the bottom of the first ledge and away from obstacles (i.e., other robots and workspace boundaries); Triangle 2's individual barrier function, $h_{t_2}(x_{t_2})$, ensures safe operation anywhere away from obstacles; and Square's individual barrier function, $h_s(x_s)$, ensures safe operation anywhere outside a distance ℓ_t from the bottom of the first and second ledges and away from obstacles. Such barrier functions can be designed using Boolean composition with nonsmooth barrier functions [38].

Triangle 1's individual and pairwise barrier functions – requiring conjunction, $\wedge$, and disjunction, $\vee$, of $h_{t_1,1}(x_{t_1}) = p_{t_1,x} + 3\ell_t$, $h_{t_1,2}(x_{t_1}) = -p_{t_1,x}$, $h_{t_1,3}(x_{t_1}) = p_{t_1,x} - \ell_t$, $h_{t_1,4}(x_{t_1}) = \ell - p_{t_1,x}$, $h_{t_1,5}(x_{t_1}) = p_{t_1,x}$, $h_{t_1,6}(x_{t_1}) = \ell_t - p_{t_1,x}$, $h_{t_1,7}(x_{t_1}) = h - p_{t_1,y}$, and $h_{t_1,8}(x_{t_1}) = p_{t_1,y}$ – are given by

$$h_{t_1}(x_{t_1}) = (h_{t_1,1}(x_{t_1}) \wedge h_{t_1,2}(x_{t_1})) \vee (h_{t_1,3}(x_{t_1}) \wedge h_{t_1,4}(x_{t_1})), \tag{32}$$

$$h_{t_1 s}(x_{t_1}, x_s) = 0, \tag{33}$$

$$h_{t_1 t_2}(x_{t_1}, x_{t_2}) = \begin{cases} (h_{t_1,5}(x_{t_1}) \wedge h_{t_1,6}(x_{t_1})) & \text{if } \psi_{t_1 t_2}(x_{t_1}, x_{t_2}) = 0 \\ \quad + (h_{t_1,7}(x_{t_1}) \wedge h_{t_1,8}(x_{t_1})), & \quad \wedge \, x_{t_1} \in \mathcal{P}_{t_2} \\ & \quad \wedge \, h_{t_1}(x_{t_1}) < 0 \\ 0, & \text{else} \end{cases}, \tag{34}$$

respectively, where $\psi_{t_1 t_2}(x_{t_1}, x_{t_2}) = p_{t_2,x}$ and $x_{t_1} \in \mathcal{P}_{t_2}$ are Triangle 1's state-dependent restriction required for pairwise collaboration with Triangle 2, interpreted as Triangle 2 remaining stationary at the first ledge while Triangle 1 climbs up it.

Triangle 2's individual and pairwise barrier functions are given by

$$h_{t_2}(x_{t_2}) = p_{t_2,x}, \tag{35}$$

$$h_{t_2 s}(x_{t_2}, x_s) = h_{t_2 t_1}(x_{t_2}, x_{t_1}) = 0. \tag{36}$$

Square's individual and pairwise barrier functions – using $h_{s,1}(x_s) = p_{s,x} + \ell$, $h_{s,2}(x_s) = -p_{s,x} - 3\ell_t$, $h_{s,3}(x_s) = p_{s,x} + 2\ell_t$, $h_{s,4}(x_s) = -p_{s,x}$, $h_{s,5}(x_s) = p_{s,x} - \ell_t$, $h_{s,6}(x_s) = \ell - p_{s,x}$, $h_{s,7}(x_s) = p_{s,x} + 3\ell_t$, $h_{s,8}(x_s) = -2\ell_t - p_{s,x}$, $h_{s,9}(x_s) = p_{s,x}$, and $h_{s,10}(x_s) = \ell_t - p_{s,x}$ – are given by

$$h_s(x_s) = (h_{s,1}(x_s) \wedge h_{s,2}(x_s)) \vee (h_{s,3}(x_s) \wedge h_{s,4}(x_s)) \vee (h_{s,5}(x_s) \wedge h_{s,6}(x_s)), \tag{37}$$

$$h_{st_1}(x_s, x_{t_1}) = \begin{cases} (h_{s,7}(x_s) \wedge h_{s,8}(x_s)) & \text{if } \psi_{st_1}(x_s, x_{t_1}) = 0 \\ \quad + (2h - p_{s,y}), & \quad \wedge x_s \in \mathcal{P}_{t_1} \\ & \quad \wedge h_s(x_s) < 0 \\ 0, & \text{else} \end{cases}, \tag{38}$$

$$h_{st_2}(x_s, x_{t_2}) = \begin{cases} (h_{s,9}(x_s) \wedge h_{s,10}(x_s)), & \text{if } \psi_{st_2}(x_s, x_{t_2}) = 0 \\ & \quad \wedge x_s \in \mathcal{P}_{t_2} \\ & \quad \wedge h_s(x_s) < 0 \\ 0, & \text{else} \end{cases}, \tag{39}$$

respectively, where $\psi_{st_1}(x_s, x_{t_1}) = p_{t_1,x} + 3\ell_t$ and $x_s \in \mathcal{P}_{t_1}$ are Square's state-dependent restrictions required for pairwise collaboration with Triangle 1, interpreted as Triangle 1 remaining stationary at the second ledge while Square climbs up it, and $\psi_{st_2}(x_s, x_{t_2}) = p_{t_2,x}$ and $x_s \in \mathcal{P}_{t_2}$ are Square's state-dependent restrictions required for pairwise collaboration with Triangle 2, interpreted as Triangle 2 remaining stationary at the first ledge while Square climbs up it.

In this scenario, $\mathcal{R}_s(x_{s,0}) \subset \bar{\mathcal{R}}_s(x_0)$, $\mathcal{R}_{t_1}(x_{t_1,0}) \subset \bar{\mathcal{R}}_{t_1}(x_0)$, and $\mathcal{R}_{t_2}(x_{t_2,0}) \subset \bar{\mathcal{R}}_{t_2}(x_0)$, which happens when Square climbs up Triangle 1 and Square and Triangle 1 climbs up Triangle 2 such that new parts of the domain become (safely) reachable to the respective robots; i.e., the individual safely reachable set has expanded for Square, Triangle 1, and Triangle 2, implying a robot mutualism – as all individuals experience a net benefit from participating in the collaborative interactions.

We provide snapshots of the simulation in Fig. 3(a)–(f). Fig. 3(a) shows the initial locations of the triangle-shaped (blue and red) and square-shaped (green) robots. Fig. 3(b) shows Triangle 1 climbing up Triangle 2 to reach the first ledge. Fig. 3(c) shows the Square climbing up Triangle 2 to reach the first ledge. Fig. 3(d) shows the Square climbing up Triangle 1 to reach the second ledge. Fig. 3(e) shows Triangle 1 climbing down Triangle 2 to reach the bottom of the first ledge. Fig. 3(f) shows the final locations of the triangular-shaped and square-shaped robots. Fig. 4(b) portrays the minimum value of each robot's pairwise barrier functions for collaboration.

B. HARDWARE EXPERIMENTS

Hardware experiments are conducted in the Robotarium [39], where users can deploy their coordinated control strategies on teams of small-scale, differential-drive robots within a 3.2 m × 2.4 m testbed. The scenarios consider robots attempting to safely reach their respective goal points after experiencing an event that causes the landscape to be comprised of water ($\mathcal{D}_{\text{water}}$; blue pixels) and land ($\mathcal{D}_{\text{land}}$; brown pixels), e.g., through flooding. There are two types of robots: a ground robot, denoted as "rabbit" (red ring), and an aquatic robot, denoted as "turtle" (green ring); the rabbit can safely operate in $\mathcal{D}_{\text{land}}$ only, whereas the turtle can safely operate in $\mathcal{D}_{\text{water}}$ only. Therefore, the rabbit requires help from the turtle to traverse $\mathcal{D}_{\text{water}}$, whereas the turtle requires help from the rabbit to traverse $\mathcal{D}_{\text{land}}$.

Each robot's states and control inputs are defined as $x_i = [p_{i,x}, p_{i,y}]^\mathsf{T} \in \mathcal{X}_i \subset \mathbb{R}^2$ and $u_i \in \mathcal{U}_i \subset \mathbb{R}^2$, respectively, where $p_{i,x}$ and $p_{i,y}$ represent the planar position states. It is also assumed that each robot exhibits single-integrator dynamics, i.e., $\dot{x}_i = u_i$, which can be realized by deriving a near-identity diffeomorphism between the desired single-integrator model and the, more accurate, unicycle model [40] (for more details, please refer to [41]).

For the experimental parameters, we let $\alpha(r) = 100r^3$; sampling period be $T = 0.033$ s; water terrain "strip" locations, with respect to x-coordinate, be $[-w_1, -w_2]$ and $[w_2, w_1]$ such that $w_1 = 0.96$ m and $w_2 = 0.32$ m; and land terrain "strip" locations, with respect to x-coordinate, be $[-l_1, -l_2]$, $[-l_3, l_3]$, and $[l_2, l_1]$ such that $l_1 = 1.6$ m, $l_2 = 0.96$ m, $l_3 = 0.32$ m.

The rabbit's individual barrier function, $h_r(x_r)$, ensures safe operation on land, whereas the turtle's individual barrier function, $h_t(x_t)$, ensures safe operation on water-based terrains.

The rabbits' individual and pairwise barrier functions – using $h_{r,1}(x_r) = x_r + l_1$, $h_{r,2}(x_r) = -l_2 - x_r$, $h_{r,3}(x_r) = x_r + l_3$, $h_{r,4}(x_r) = l_3 - x_r$, $h_{r,5}(x_r) = x_r - l_2$, $h_{r,6}(x_r) = x_r + l_1$ to define the three land strips – are given by

$$h_r(x_r) = (h_{r,1}(x_r) \wedge h_{r,2}(x_r)) \vee (h_{r,3}(x_r) \wedge h_{r,4}(x_r)) \vee (h_{r,5}(x_r) \wedge h_{r,6}(x_r)), \tag{40}$$

$$h_{rt}(x_r, x_t) = \begin{cases} -h_r(x_r), & \text{if } \psi_{rt}(x_r, x_t) = 0 \wedge \\ & \quad h_r(x_r) < 0 \\ 0, & \text{else} \end{cases}, \tag{41}$$

respectively, where $\psi_{rt}(x_r, x_t) = \|x_r - x_t\|_2$ is the rabbit's state-dependent restriction required for pairwise collaboration with a turtle, interpreted as the turtle carrying the rabbit.

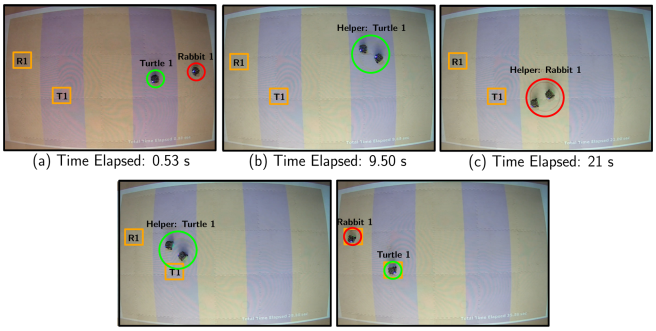
The turtles' individual and pairwise barrier functions – using $h_{t,1}(x_t) = x_t + w_1$, $h_{t,2}(x_t) = -x_t - w_2$, $h_{t,3}(x_t) = x_t - w_2$, $h_{t,4}(x_t) = w_1 - x_t$ to define the two water strips – are given by

$$h_t(x_t) = (h_{t,1}(x_t) \wedge h_{t,2}(x_t)) \vee (h_{t,3}(x_t) \wedge h_{t,4}(x_t)), \tag{42}$$

$$h_{tr}(x_t, x_r) = \begin{cases} -h_t(x_t), & \text{if } \psi_{tr}(x_t, x_r) = 0 \wedge \\ & \quad h_t(x_t) < 0 \\ 0, & \text{else} \end{cases}, \tag{43}$$

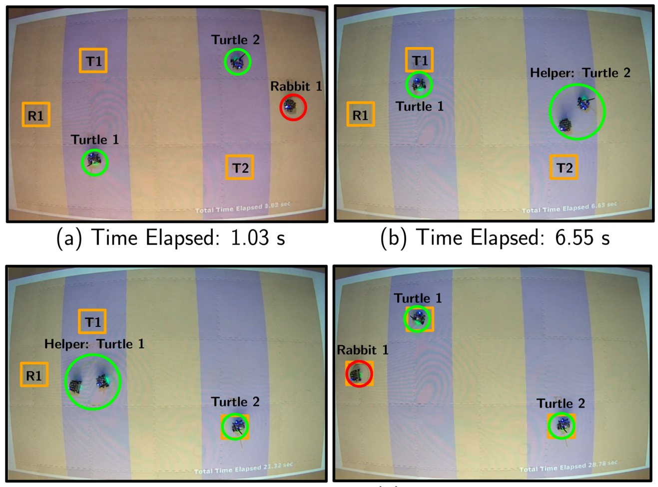
respectively, where $\psi_{tr}(x_t, x_r) = \|x_t - x_r\|_2$ is the turtle's state-dependent restriction required for pairwise collaboration with a rabbit, interpreted as the rabbit carrying the turtle.

The mobile robots contained in the Robotarium have collision avoidance barrier function constraints imposed on them [39]. Therefore, the turtles cannot physically transport the rabbit across $\mathcal{D}_{\text{water}}$, and, likewise, the rabbits cannot physically transport the turtle across $\mathcal{D}_{\text{land}}$. Thus, "carrying", i.e.,



(a) Time Elapsed: 0.53 s (b) Time Elapsed: 9.50 s (c) Time Elapsed: 21 s (d) Time Elapsed: 29.36 s (e) Time Elapsed: 35.48 s

FIGURE 5. Scenario 1: Robot mutualism between one ground robot (denoted as "rabbit"; red ring) and one aquatic robot (denoted as "turtle"; green ring) tasked with safely reaching their respective goal points (orange squares). [Supplemental Video: https://youtu.be/Q062xDkw140].



(a) Time Elapsed: 1.03 s (b) Time Elapsed: 6.55 s (c) Time Elapsed: 21.32 s (d) Time Elapsed: 28.78 s

FIGURE 6. Scenario 2: Robot commensalism between one rabbit and two turtles. [Supplemental Video: https://youtu.be/sO_GKFHkYG0].

help, is depicted as when a turtle and rabbit move together in close proximity during collaboration, with the help-providing robot trailing behind the help-receiving robot.

1) SCENARIO 1: GROUND-AQUATIC ROBOTS

We consider one rabbit and one turtle, where both the rabbit's and turtle's individual safely reachable set can expand from helping each other – i.e., $\mathcal{R}_{r_1}(x_{r_1,0}) \subset \bar{\mathcal{R}}_{r_1}(x_0)$ and $\mathcal{R}_{t_1}(x_{t_1,0}) \subset \bar{\mathcal{R}}_{t_1}(x_0)$, which happens when the rabbit "carries" the turtle on land-based terrain and the turtle "ferries" the rabbit on water-based terrains such that the entire domain becomes (safely) reachable to both the rabbit and turtle, implying a robot mutualism. Also, by construction, at some point in time, we have $x_{r_1} \in \mathcal{R}_{t_1}(x_{t_1,0})$ and $x_{t_1} \in \mathcal{R}_{r_1}(x_{r_1,0})$ to make collaboration feasible.

We provide snapshots of the hardware experiment in Fig. 5(a)–(d). Fig. 5(a) shows the initial locations of Rabbit 1 (red ring) and Turtle 1 (green ring). Fig. 5(b) shows Turtle 1 "ferrying" Rabbit 1 across the water. Fig. 5(c) shows Rabbit 1 "carrying" Turtle 1 across the land. Fig. 5(d) shows the final locations of Rabbit 1 and Turtle 1. Although it is not shown, $H_i^{\min}(x(t)) \geq 0 \; \forall t$ (as in Fig. 4).

2) SCENARIO 2: GROUND-AQUATIC ROBOTS

We consider one rabbit and two turtles, where $\mathcal{R}_{r_1}(x_{r_1,0}) \subset \bar{\mathcal{R}}_{r_1}(x_0)$, $\mathcal{R}_{t_1}(x_{t_1,0}) = \bar{\mathcal{R}}_{t_1}(x_0)$, and $\mathcal{R}_{t_2}(x_{t_2,0}) = \bar{\mathcal{R}}_{t_2}(x_0)$, which happens when the turtle "ferries" the rabbit on water-based terrains such that the entire domain becomes (safely) reachable to the rabbit, implying a robot commensalism.

We provide snapshots of the hardware experiment in Fig. 6(a)–(d). Fig. 6(a) shows the initial locations of Rabbit 1, Turtle 1, and Turtle 2. Fig. 6(b) shows Turtle 2 "ferrying" Rabbit 1 across the water, while Turtle 1 progresses toward its goal point. Fig. 6(c) shows Turtle 1 "ferrying" Rabbit 1 the water, while Turtle 2 has reached its goal point. Fig. 6(d) shows the final locations of Rabbit 1, Turtle 1, and Turtle 2. Also, note that $H_i^{\min}(x(t)) \geq 0 \; \forall t$.

V. CONCLUSION

In this paper, we addressed the questions of "Why?", "When?", and "How?" a team of heterogeneous robots should collaborate. We first investigated why it would be beneficial for robots to form collaborative arrangements through comparison of safe sets and safely reachable sets. We then provided the conditions that must hold – in terms of barrier functions and safely reachable sets – between suitably paired help-receiving and help-providing robots such that they can achieve and evolve on their collaboration submanifold. We also described the pairwise collaboration framework that ensures robots always remain safe, whether they collaborate or not. Lastly, we demonstrated the pairwise collaboration framework in case studies on a team of mobile robots numerically and experimentally.

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ALEXANDER A. NGUYEN (Graduate Student Member, IEEE) received the B.S. degree in mechanical engineering from the University of California, Santa Barbara, CA, USA, and the M.S. degree in mechanical and aerospace engineering from the University of California, Irvine, CA, USA. He is currently working toward the Ph.D. degree with Mechanical and Aerospace Engineering Department, University of California, Irvine. His research interests include control theory, optimization, and state estimation with applications to heterogeneous multi-robot systems.

FARYAR JABBARI (Life Senior Member, IEEE) is currently on the faculty of the Mechanical and Aerospace Engineering Department, University of California, Irvine, CA, USA. His research focuses on control theory and its applications. He was an Associate Editor for *Automatica* and IEEE TRANSACTIONS ON AUTOMATIC CONTROL, and also the Program Chair for ACC-11 and CDC-09, and the General Chair for CDC-14.

MAGNUS EGERSTEDT (Fellow, IEEE) received the B.A. degree in Philosophy from Stockholm University, Stockholm, the M.S. degree in engineering physics, and the Ph.D. degree in applied mathematics from the Royal Institute of Technology, Stockholm, Sweden. He is currently the Dean of Engineering with the University of California, Irvine, CA, USA. He was on the faculty at the Georgia Institute of Technology, Atlanta, GA, USA. His research interests include control theory and robotics. Dr. Egerstedt is the President of the IEEE Control Systems Society. He has received several teaching and research awards, and is a Fellow of IFAC and the Royal Swedish Academy of Engineering Science.